1998 AP Calculus BC: Section I, Part A

55 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

(A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers

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2. In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

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3. The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2, -1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

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4. $\int \frac{1}{x^2 - 6x + 8}\,dx =$

(A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

(B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

(C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right| + C$

(D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right| + C$

(E) $\ln\left|(x-2)(x-4)\right| + C$

5. If f and g are twice differentiable and if $h(x) = f\left(g(x)\right)$, then $h''(x) =$

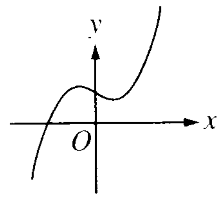
(A) $f''\left(g(x)\right)\left[g'(x)\right]^2 + f'\left(g(x)\right)g''(x)$

(B) $f''\left(g(x)\right)g'(x) + f'\left(g(x)\right)g''(x)$

(C) $f''\left(g(x)\right)\left[g'(x)\right]^2$

(D) $f''\left(g(x)\right)g''(x)$

(E) $f''\left(g(x)\right)$



6. The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

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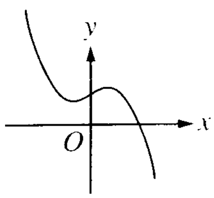

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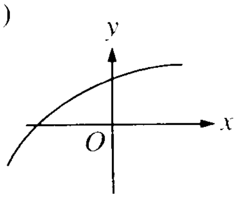

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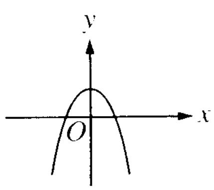

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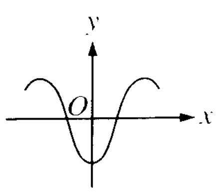
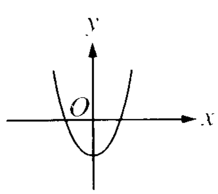


(E)


7. $\int_1^e \left(\frac{x^2-1}{x}\right) dx =$

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\frac{e^2}{2}-\frac{3}{2}$

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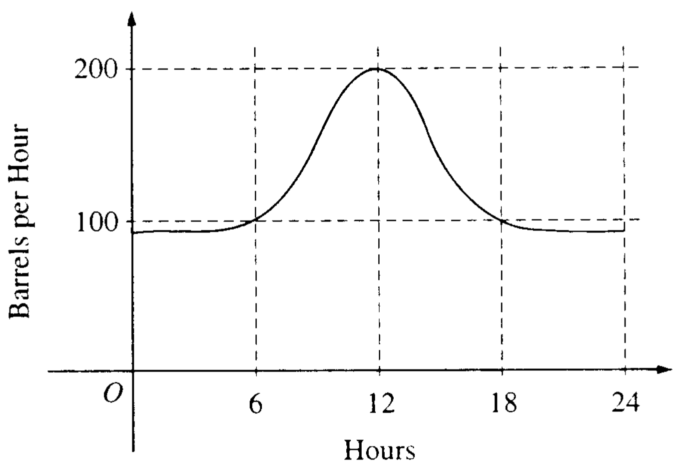
8. If $\frac{dy}{dx}=\sin x\cos^2 x$ and if $y=0$ when $x=\frac{\pi}{2}$, what is the value of y when $x=0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

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10. A particle moves on a plane curve so that at any time $t>0$ its x-coordinate is t^3-t and its y-coordinate is $(2t-1)^3$. The acceleration vector of the particle at $t=1$ is

(A) $(0,1)$ (B) $(2,3)$ (C) $(2,6)$ (D) $(6,12)$ (E) $(6,24)$

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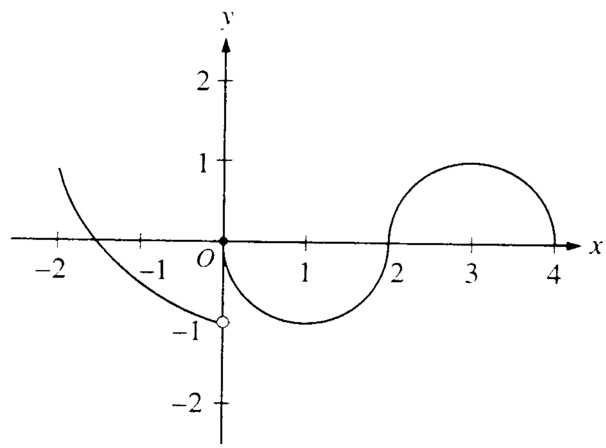
11. If f is a linear function and $0<a<b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b-a$ (E) $\frac{b^2-a^2}{2}$

12. If $f(x)=\begin{cases} \ln x & \text{for } 0<x\le 2 \\ x^2\ln 2 & \text{for } 2<x\le 4, \end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2<x<4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

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14. What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x=0$ for $\sin x$?

(A) $1-\frac{1}{2}+\frac{1}{24}$

(B) $1-\frac{1}{2}+\frac{1}{4}$

(C) $1-\frac{1}{3}+\frac{1}{5}$

(D) $1-\frac{1}{4}+\frac{1}{8}$

(E) $1-\frac{1}{6}+\frac{1}{120}$

15. $\int x \cos x \, dx =$

(A) $x \sin x - \cos x + C$

(B) $x \sin x + \cos x + C$

(C) $-x \sin x + \cos x + C$

(D) $x \sin x + C$

(E) $\frac{1}{2}x^2 \sin x + C$

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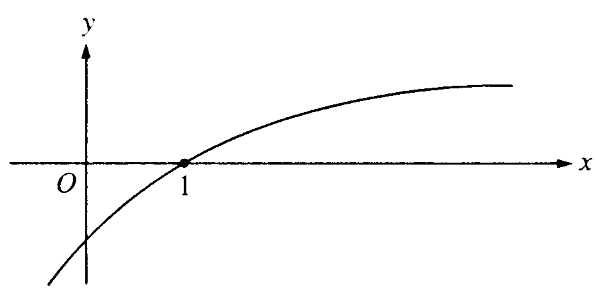
16. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

(A) -1 (B) 0 (C) 1 (D) 0 and 1 (E) -1, 0, and 1



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

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19. The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve $r = 2$ is given by

(A) $\frac{1}{2}\int_0^{\pi} (4\sin\theta - 2)^2\, d\theta$ (B) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4\sin\theta - 2)^2\, d\theta$ (C) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4\sin\theta - 2)^2\, d\theta$

(D) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(16\sin^2\theta - 4\right) d\theta$ (E) $\frac{1}{2}\int_0^{\pi} \left(16\sin^2\theta - 4\right) d\theta$

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20. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k ?

(A) 3   (B) 4   (C) 6   (D) 8   (E) 12

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21. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

(A) $\int_0^1 \sqrt{t^2 + 1}\, dt$

(B) $\int_0^1 \sqrt{t^2 + t}\, dt$

(C) $\int_0^1 \sqrt{t^4 + t^2}\, dt$

(D) $\frac{1}{2}\int_0^1 \sqrt{4 + t^4}\, dt$

(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9}\, dt$

22. If $\lim_{b\to\infty}\int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges

(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

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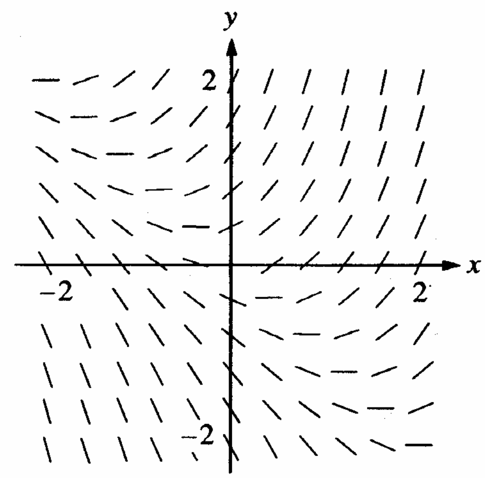
23. Let f be a function defined and continuous on the closed interval $[a,b]$. If f has a relative maximum at c and $a < c < b$, which of the following statements must be true?

I. $f'(c)$ exists.
II. If $f'(c)$ exists, then $f'(c) = 0$.
III. If $f''(c)$ exists, then $f''(c) \le 0$.

(A) II only (B) III only (C) I and II only (D) I and III only (E) II and III only



24. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

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25. $\int_0^{\infty} x^2 e^{-x^3}\, dx$ is

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

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26. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t \to \infty} P(t)$?

(A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

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27. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n{}^{n-1}$

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28. $\lim_{x \to 1} \frac{\int_1^x e^{t^2}\, dt}{x^2 - 1}$ is

(A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

1998 AP Calculus BC: Section I, Part B

50 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. For what integer k, $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

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77. If f is a vector-valued function defined by $f(t) = \left(e^{-t}, \cos t\right)$, then $f''(t) =$

(A) $-e^{-t} + \sin t$
(B) $e^{-t} - \cos t$
(C) $\left(-e^{-t}, -\sin t\right)$
(D) $\left(e^{-t}, \cos t\right)$
(E) $\left(e^{-t}, -\cos t\right)$

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78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

79. Let f be the function given by $f(x)=\dfrac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers x?

(A) None
(B) 1 only
(C) 2 only
(D) 4 only
(E) 1 and 4 only

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80. Let R be the region enclosed by the graph of $y=1+\ln\left(\cos^4 x\right)$, the x-axis, and the lines $x=-\dfrac{2}{3}$ and $x=\dfrac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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81. If $\dfrac{dy}{dx}=\sqrt{1-y^2}$, then $\dfrac{d^2y}{dx^2}=$

(A) $-2y$ (B) $-y$ (C) $\dfrac{-y}{\sqrt{1-y^2}}$ (D) y (E) $\dfrac{1}{2}$

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82. If $f(x)=g(x)+7$ for $3\le x\le 5$, then $\int_3^5 \left[f(x)+g(x)\right]dx=$

(A) $2\int_3^5 g(x)\,dx+7$

(B) $2\int_3^5 g(x)\,dx+14$

(C) $2\int_3^5 g(x)\,dx+28$

(D) $\int_3^5 g(x)\,dx+7$

(E) $\int_3^5 g(x)\,dx+14$

83. The Taylor series for $\ln x$, centered at $x = 1$, is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

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84. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

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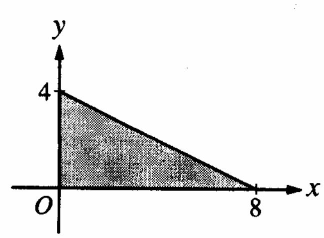
x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5]$, $[5, 7]$, and $[7, 8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

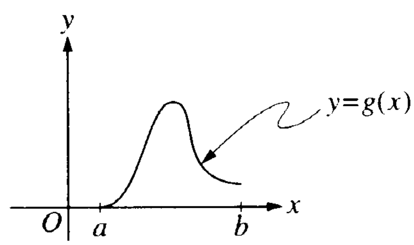


86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

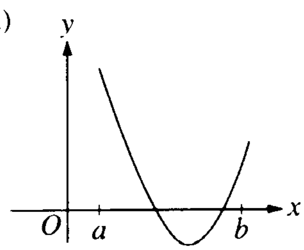
(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$



88. Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a, b]$. Which of the following could be the graph of f on $[a, b]$?

(A)


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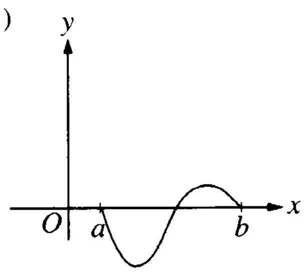
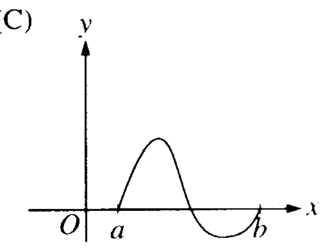


(C)


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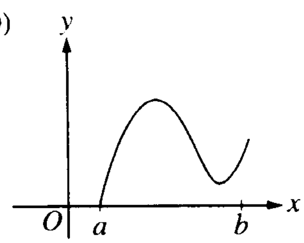
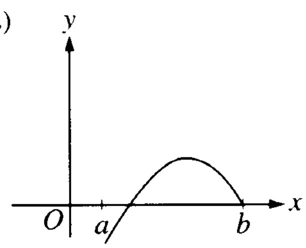


(E)


89. The graph of the function represented by the Maclaurin series

$$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^n}{n!} + \ldots$$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

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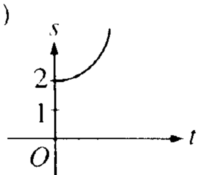
90. A particle starts from rest at the point $(2,0)$ and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t?

(A)


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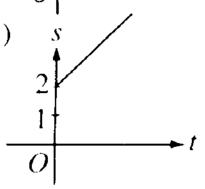
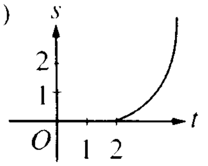


(C)


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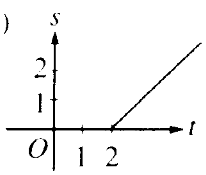
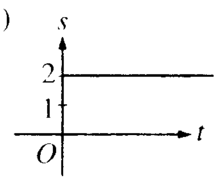


E)


t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

91. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

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92. Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

(A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8

1998 Calculus BC Solutions: Part A

1. C f will be increasing when its derivative is positive.
$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$ $f'(x) = 3(x+3)(x-1) > 0$ for $x < -3$ or $x > 1$.

2. A $\frac{dx}{dt} = 5$ and $\frac{dy}{dt} = 3 \Rightarrow \frac{dy}{dx} = \frac{3}{5}$

3. D Find the derivative implicitly and substitute. $2y \cdot y' + 3(xy+1)^2(x \cdot y' + y) = 0$;
$2(-1) \cdot y' + 3((2)(-1)+1)^2((2) \cdot y' + (-1)) = 0$; $-2y' + 6 \cdot y' - 3 = 0$; $y' = \frac{3}{4}$

4. A Use partial fractions. $\frac{1}{x^2 - 6x + 8} = \frac{1}{(x-4)(x-2)} = \frac{1}{2}\left(\frac{1}{x-4} - \frac{1}{x-2}\right)$

$$\int \frac{1}{x^2 - 6x + 8}\,dx = \frac{1}{2}\left(\ln|x-4| - \ln|x-2|\right) + C = \frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$$

5. A $h'(x) = f'\left(g(x)\right) \cdot g'(x)$; $h''(x) = f''\left(g(x)\right) \cdot g'(x) \cdot g'(x) + f'\left(g(x)\right) \cdot g''(x)$
$h''(x) = f''\left(g(x)\right) \cdot \left(g'(x)\right)^2 + f'\left(g(x)\right) \cdot g''(x)$

6. E The graph of h has 2 turning points and one point of inflection. The graph of h' will have 2 x-intercepts and one turning point. Only (C) and (E) are possible answers. Since the first turning point on the graph of h is a relative maximum, the first zero of h' must be a place where the sign changes from positive to negative. This is option (E).

7. E $\int_1^e \frac{x^2 - 1}{x}\,dx = \int_1^e x - \frac{1}{x}\,dx = \left(\frac{1}{2}x^2 - \ln x\right)\Big|_1^e = \left(\frac{1}{2}e^2 - 1\right) - \left(\frac{1}{2} - 0\right) = \frac{1}{2}e^2 - \frac{3}{2}$

8. B $y(x) = -\frac{1}{3}\left(\cos x\right)^3 + C$; Let $x = \frac{\pi}{2}$, $0 = -\frac{1}{3}\left(\cos\frac{\pi}{2}\right)^3 + C \Rightarrow C = 0$. $y(0) = -\frac{1}{3}\left(\cos 0\right)^3 = -\frac{1}{3}$

9. D Let $r(t)$ be the rate of oil flow as given by the graph, where t is measured in hours. The total number of barrels is given by $\int_0^{24} r(t)dt$. This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about $3{,}000$ barrels.

10. E $v(t) = \left(3t^2 - 1, 6(2t-1)^2\right)$ and $a(t) = \left(6t, 24(2t-1)\right) \Rightarrow a(1) = (6, 24)$

11. A Since f is linear, its second derivative is zero and the integral gives the area of a rectangle with zero height and width $(b-a)$. This area is zero.

12. E $\lim\limits_{x\to 2^-} f(x) = \ln 2 \neq 4\ln 2 = \lim\limits_{x\to 2^+} f(x)$. Therefore the limit does not exist.

13. B At $x = 0$ and $x = 2$ only. The graph has a non-vertical tangent line at every other point in the interval and so has a derivative at each of these other x's.

14. E $\sin x \approx x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$; $\sin 1 \approx 1 - \dfrac{1^3}{3!} + \dfrac{1^5}{5!} = 1 - \dfrac{1}{6} + \dfrac{1}{120}$

15. B Use the technique of antiderivatives by parts. Let $u = x$ and $dv = \cos x\,dx$.

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$$

16. C Inflection point will occur when f'' changes sign. $f'(x) = 15x^4 - 20x^3$.

$f''(x) = 60x^3 - 60x^2 = 60x^2(x-1)$. The only sign change is at $x = 1$.

17. D From the graph $f(1) = 0$. Since $f'(1)$ represents the slope of the graph at $x = 1$, $f'(1) > 0$. Also, since $f''(1)$ represents the concavity of the graph at $x = 1$, $f''(1) < 0$.

18. B
I. Divergent. The limit of the nth term is not zero.
II. Convergent. This is the same as the alternating harmonic series.
III. Divergent. This is the harmonic series.

19. D Find the points of intersection of the two curves to determine the limits of integration.

$4\sin\theta = 2$ when $\sin\theta = 0.5$; this is at $\theta = \dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$. Area $= \dfrac{1}{2}\displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left((4\sin\theta)^2 - (2)^2\right) d\theta$

20. E $$\left.\frac{d\left(\sqrt[3]{x}\right)}{dt}\right|_{x=8} = \left.\frac{1}{3}x^{-\frac{2}{3}}\cdot\frac{dx}{dt}\right|_{x=8} = \frac{1}{3}(8)^{-\frac{2}{3}}\cdot\frac{dx}{dt} = \frac{1}{3}\cdot\frac{1}{4}\cdot\frac{dx}{dt} = \frac{1}{12}\cdot\frac{dx}{dt} \Rightarrow k = 12$$

21 C The length of this parametric curve is given by $\displaystyle\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{(t^2)^2 + t^2}\,dt$.

22. A This is the integral test applied to the series in (A). Thus the series in (A) converges. None of the others must be true.

23. E I. False. The relative maximum could be at a cusp.
II. True. There is a critical point at $x = c$ where $f'(c)$ exists
III. True. If $f''(c) > 0$, then there would be a relative minimum, not maximum

24. C All slopes along the diagonal $y = -x$ appear to be 0. This is consistent only with option (C). For each of the others you can see why they do not work. Option (A) does not work because all slopes at points with the same x coordinate would have to be equal. Option (B) does not work because all slopes would have to be positive. Option (D) does not work because all slopes in the third quadrant would have to be positive. Option (E) does not work because there would only be slopes for $y > 0$.

25. C $\int_0^\infty x^2 e^{-x^3}\,dx = \lim_{b\to\infty} \int_0^b x^2 e^{-x^3}\,dx = \lim_{b\to\infty} -\frac{1}{3}e^{-x^3}\Big|_0^b = \frac{1}{3}$.

26. E As $\lim_{t\to\infty} \frac{dP}{dt} = 0$ for a population satisfying a logistic differential equation, this means that $P \to 10{,}000$.

27. D If $f(x) = \sum_{n=0}^{\infty} a_n x^n$, then $f'(x) = \sum_{n=0}^{\infty} n a_n x^{n-1} = \sum_{n=1}^{\infty} n a_n x^{n-1}$.

$f'(1) = \sum_{n=1}^{\infty} n a_n 1^{n-1} = \sum_{n=1}^{\infty} n a_n$

28. C Apply L'Hôpital's rule. $\lim_{x\to 1} \frac{\int_1^x e^{t^2}\,dt}{x^2 - 1} = \lim_{x\to 1} \frac{e^{x^2}}{2x} = \frac{e}{2}$

76. D The first series is either the harmonic series or the alternating harmonic series depending on whether k is odd or even. It will converge if k is odd. The second series is geometric and will converge if $k < 4$.

77. E $f'(t) = \left(-e^{-t}, -\sin t\right)$; $f''(t) = \left(e^{-t}, -\cos t\right)$.

78. B $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. However, $C = 2\pi r$ and $\frac{dr}{dt} = -0.1$. Thus $\frac{dA}{dt} = -0.1C$.

79. A None. For every positive value of a the denominator will be zero for some value of x.

80. B The area is given by $\int_{-\frac{2}{3}}^{\frac{2}{3}} \left(1 + \ln(\cos^4 x)\right) dx = 0.919$

81. B $\frac{dy}{dx} = \sqrt{1 - y^2}$; $\frac{d^2 y}{dx^2} = \frac{d}{dx}\left((1 - y^2)^{\frac{1}{2}}\right) = \frac{1}{2}\left(1 - y^2\right)^{-\frac{1}{2}} \cdot (-2y) \cdot \frac{dy}{dx} = -y$

82. B $\int_3^5 \left[f(x) + g(x)\right] dx = \int_3^5 \left[2g(x) + 7\right] dx = 2\int_3^5 g(x)\,dx + (7)(2) = 2\int_3^5 g(x)\,dx + 14$

83. C Use a calculator. The maximum for $\left|\ln x - \left(\frac{(x-1)}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}\right)\right|$ on the interval $0.3 \le x \le 1.7$ occurs at $x = 0.3$.

84. B You may use the ratio test. However, the series will converge if the numerator is $(-1)^n$ and diverge if the numerator is 1^n. Any value of x for which $|x + 2| > 1$ in the numerator will make the series diverge. Hence the interval is $-3 \le x < -1$.

85. C There are 3 trapezoids. $\frac{1}{2} \cdot 3\left(f(2) + f(5)\right) + \frac{1}{2} \cdot 2\left(f(5) + f(7)\right) + \frac{1}{2} \cdot 1\left(f(7) + f(8)\right)$

86. C Each cross section is a semicircle with a diameter of y. The volume would be given by $\int_0^8 \frac{1}{2}\pi\left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8}\int_0^8 \left(\frac{8-x}{2}\right)^2 dx = 16.755$

87. D Find the x for which $f'(x)=1$. $f'(x)=4x^3+4x=1$ only for $x=0.237$. Then $f(0.237)=0.115$. So the equation is $y-0.115=x-0.237$. This is equivalent to option (D).

88. C From the given information, f is the derivative of g. We want a graph for f that represents the slopes of the graph g. The slope of g is zero at a and b. Also the slope of g changes from positive to negative at one point between a and b. This is true only for figure (C).

89. A The series is the Maclaurin expansion of e^{-x}. Use the calculator to solve $e^{-x}=x^3$.

90. A Constant acceleration means linear velocity which in turn leads to quadratic position. Only the graph in (A) is quadratic with initial $s=2$.

91. E $v(t)=11+\int_0^t a(x)\,dx \approx 11+\left[2\cdot 5+2\cdot 2+2\cdot 8\right]=41$ ft/sec.

92. D $f'(x)=2x-2,\ f'(2)=2,$ and $f(2)=3$, so an equation for the tangent line is $y=2x-1$. The difference between the function and the tangent line is represented by $(x-2)^2$. Solve $(x-2)^2<0.5$. This inequality is satisfied for all x such that $2-\sqrt{0.5}<x<2+\sqrt{0.5}$. This is the same as $1.293<x<2.707$. Thus the largest value in the list that satisfies the inequality is 2.7.