

Review Packet: Precalculus

Name _____

Period _____

Precalculus Semester 2 Mastery #1a)

- 1.) Given $w(q(x)) = f(x)$ find two possibilities for $w(x)$ and $q(x)$ where neither is equal to x .

$$f(x) = \sin^2(3x) - 7$$

$$w(x) = \underline{\hspace{2cm}} \quad w(x) = \underline{\hspace{2cm}}$$

$$q(x) = \underline{\hspace{2cm}} \quad q(x) = \underline{\hspace{2cm}}$$

- 2.) Expand the following.

$$\log_4\left(\frac{xy^3}{16z}\right)$$

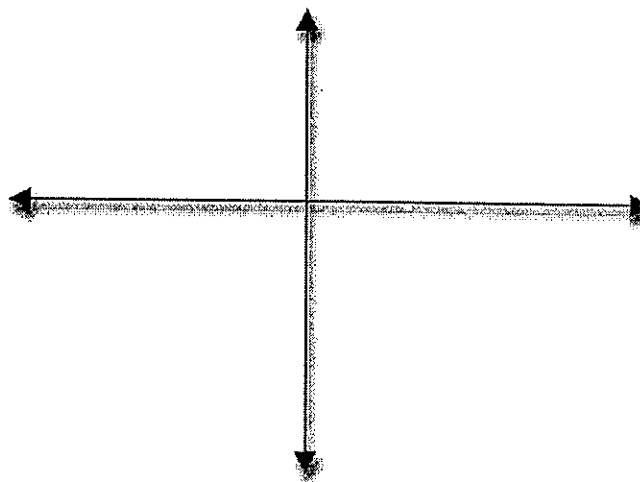
- 3.) Sketch a graph of the following. Find the intercepts and end behavior.

$$f(x) = -6x^3 - 17x^2 + 5x + 6$$

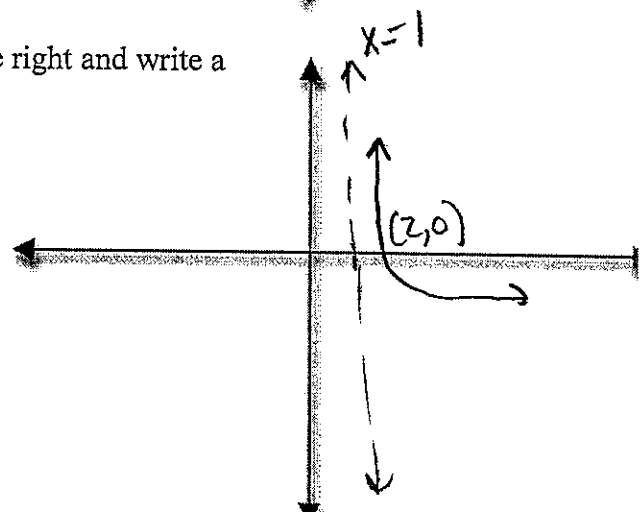
zeros:

y-intercept:

End behavior in limit notation



- 4.) Give the end behavior for the graph shown to the right and write a possible equation for the graph.



5.) Evaluate the value of each of the following. **Show the coordinate on the unit circle or draw the triangle.**

a.) $\cos\left(\frac{7\pi}{6}\right)$

b.) $\tan\left(\frac{2\pi}{3}\right)$

c.) $\csc\left(\frac{5\pi}{4}\right)$

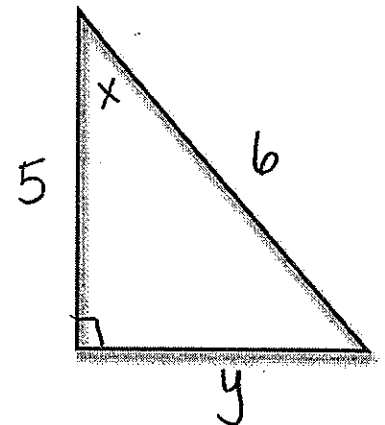
d.) $\cot\left(-\frac{7\pi}{4}\right)$

e.) $\sin\left(\frac{11\pi}{3}\right)$

f.) $\sec(\pi)$

6.) Find the $\tan \theta$ if $\sin \theta = \frac{2}{3}$ and $\tan \theta < 0$.

7.) Give the exact value for x and y shown in the diagram below.



8.) Find all value for $0 \leq \theta \leq 2\pi$ where $\sin \theta = \frac{\sqrt{3}}{2}$. Show how you found your solutions.

Name _____
Period _____

Mastery #2 a.) Precalculus

Solve the following equations. Give the exact solutions. Give logarithmic solutions in terms of \ln .

1.) $7x^3 - 12x^2 = 4x$

2.) $4\left(\frac{1}{3}\right)^{2x-4} + 5 = 8$

3.) $\frac{1}{x} - \frac{5}{x+1} = 3$

4.) $(x-1)^{\frac{3}{2}} = -8$

5.) $6\sin(3x) - \sqrt{27} = 0$

6.) Find three different decompositions for the following where $q(k(x)) = h(x)$.

$$h(x) = \frac{5}{(4x+1)^2} - 3$$

7.) Evaluate the following. Give only the FUNCTION ANSWER!

a.) $\tan\left(-\frac{7\pi}{6}\right)$

b.) $\sin^{-1}(-1)$

c.) $\cos\left(\arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$

8.) Add the following- Find a common denominator.

$$\frac{1}{x+h} - \frac{1}{x}$$
$$\frac{\quad}{x+h-x}$$

Name _____
Period _____

Mastery #3a.)

- 1.) The volume of a box is given by the function $V(x) = x(10 - 2x)(14 - 2x)$, where x is the height of the box. For what heights will the box have a volume of 25 cubic units.

- 2.) Solve the equation. Find ALL solutions.

$$4 \sin(2x) - \sqrt{8} = 0$$

- 3.) Sketch a graph of $f(x) = \sqrt{x}$. Find the average rate of change between $x = 1$ and $x = 1 + h$. Find the conjugate to estimate the tangent line slope at $x = 1$.

4.) Sketch a graph of $f(x) = x \sin x$ on the interval $[0, 2\pi]$. Find the following:

Increasing intervals

Decreasing intervals

Absolute mins

Relative mins

Absolute Maxs

Relative maxs

5.) Solve the equation: $\ln x - \ln(x - 1) = 2$

Name _____

Period _____

Mastery 4a.) Summative Assessment

1.) Condense the logarithm and then sketch a graph.

$$f(x) = \ln(2x + 2) - \ln 2$$

x-intercept

y-intercept

Domain

Range

End Behavior in Limit Notation

2.) Sketch a graph of $f(x) = \frac{x^2}{x-1}$

End Behavior Asymptote

x-intercept

y-intercept

3.) The perimeter of rectangle is 100 feet. Set up a function that models the area of the rectangle. Sketch a graph of your model and find the maximum area possible.

4.) Sketch a graph of the trigonometric function.

$$f(x) = 5 \tan(2x) + 5$$

Transformations

y-intercept

x-intercept

Domain

Range

5.) Verify the identity:

$$\frac{\sec x + 1}{\tan x} = \frac{\sin x}{1 - \cos x}$$

Name Key Period _____

Precalculus Semester 2 Mastery #1a)

- 1.) Given $w(q(x)) = f(x)$ find two possibilities for $w(x)$ and $q(x)$ where neither is equal to x .

$$f(x) = \sin^2(3x) - 7$$

Answers vary

$$w(x) = \underline{\sin^2 x - 7} \quad w(x) = \underline{x^2 - 7}$$

$$q(x) = \underline{3x} \quad q(x) = \underline{\sin(3x)}$$

- 2.) Expand the following.

$$\log_4 \left(\frac{xy^3}{16z} \right) = \log_4 x + 3\log_4 y - 2 - \log_4 z$$

- 3.) Sketch a graph of the following. Find the intercepts and end behavior.

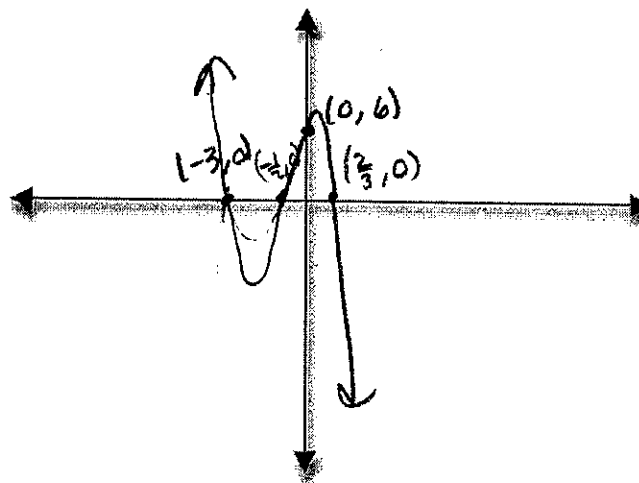
$$f(x) = -6x^3 - 17x^2 + 5x + 6$$

$$\begin{array}{r|rrrr} 3 & -6 & -17 & 5 & 6 \\ & & 18 & -3 & -6 \\ \hline & -6 & 1 & 2 & 0 \end{array}$$

zeros: $(-3, 0)$ $(\frac{2}{3}, 0)$ $(-\frac{1}{2}, 0)$

$$\begin{aligned} -6x^2 + x + 2 &= 0 \\ -3x + 2)(2x + 1) &= 0 \\ x = +\frac{2}{3} \quad x = -\frac{1}{2} \end{aligned}$$

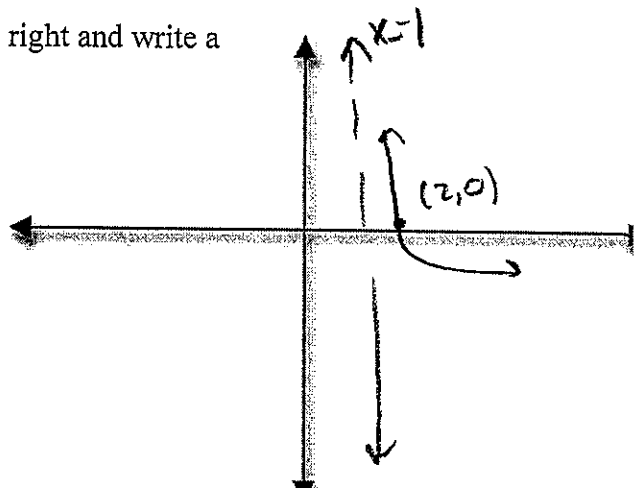
y-intercept: $(0, 6)$ $\lim_{x \rightarrow \infty} f(x) = -\infty$
 $\lim_{x \rightarrow -\infty} f(x) = \infty$
 End behavior in limit notation



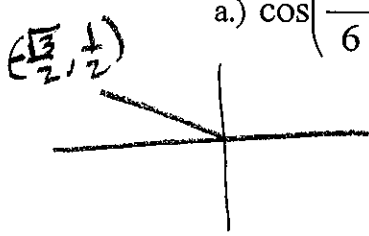
- 4.) Give the end behavior for the graph shown to the right and write a possible equation for the graph.

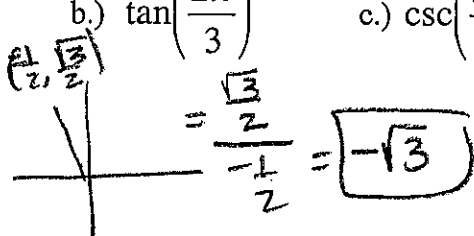
$$y = \log_{\frac{1}{2}}(x-1)$$

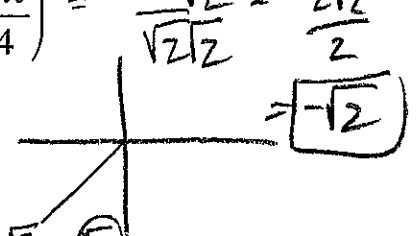
$$\begin{aligned} \lim_{x \rightarrow \infty} f(x) &= -\infty \\ \lim_{x \rightarrow 1^+} f(x) &= \infty \end{aligned}$$

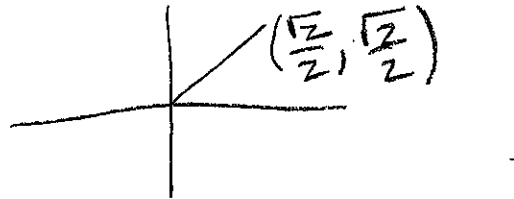


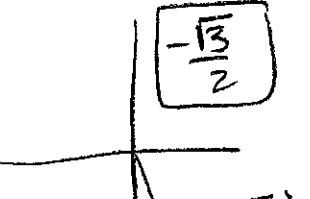
5.) Evaluate the value of each of the following. Show the coordinate on the unit circle or draw the triangle.

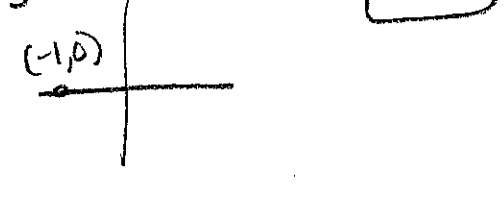
a.) $\cos\left(\frac{7\pi}{6}\right) = \boxed{-\frac{\sqrt{3}}{2}}$ 

b.) $\tan\left(\frac{2\pi}{3}\right) = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \boxed{-\sqrt{3}}$ 

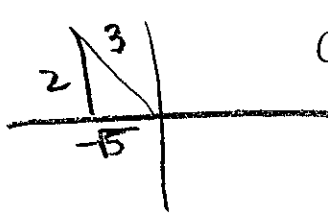
c.) $\csc\left(\frac{5\pi}{4}\right) = \frac{-2\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{-2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$ 

d.) $\cot\left(-\frac{7\pi}{4}\right) = \boxed{1}$ 

e.) $\sin\left(\frac{11\pi}{3}\right) = \sin\left(\frac{5\pi}{3}\right) = \boxed{-\frac{\sqrt{3}}{2}}$ 

f.) $\sec(\pi) = \frac{1}{\cos(\pi)} = \frac{1}{-1} = \boxed{-1}$ 

6.) Find the $\tan\theta$ if $\sin\theta = \frac{2}{3}$ and $\tan\theta < 0$.



$a^2 + 4 = 9$
 $a^2 = 5$
 $a = \pm\sqrt{5}$

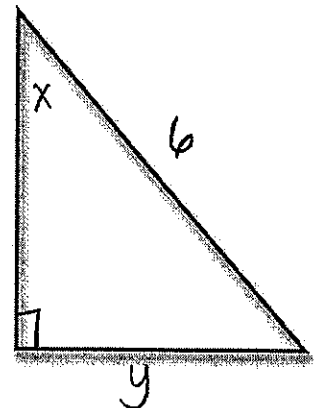
$\tan\theta = \frac{2\sqrt{5}}{-\sqrt{5}\sqrt{5}} = \boxed{-\frac{2\sqrt{5}}{5}}$

7.) Give the exact value for x and y shown in the diagram below.

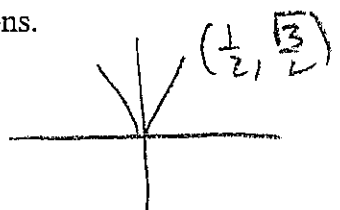
$5^2 + y^2 = 6^2$ $\cos^{-1}\left(\frac{5}{6}\right) = \cos^{-1}\left(\frac{5}{6}\right)$

$25 + y^2 = 36$
 $-25 \quad -25$
 $y^2 = 11$
 $y = \pm\sqrt{11}$
 $y = \sqrt{11}$

$x = \cos^{-1}\left(\frac{5}{6}\right)$



8.) Find all value for $0 \leq \theta \leq 2\pi$ where $\sin\theta = \frac{\sqrt{3}}{2}$. Show how you found your solutions.



$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

Mastery #2 a.) Precalculus

Solve the following equations. Give the exact solutions. Give logarithmic solutions in terms of \ln .

1.) $7x^3 - 12x^2 = 4x$

$$7x^3 - 12x^2 - 4x = 0$$

$$x(7x^2 - 12x - 4) = 0$$

$$x(7x + 2)(x - 2) = 0$$

$$x = 0 \quad x = -\frac{2}{7} \quad x = 2$$

3.) $\frac{1^x(x+1)}{x} - \frac{5(x)(x+1)}{x+1} = 3(x)(x+1)$

$$x+1 - 5x = 3x^2 + 3x$$

$$-4x + 1 = 3x^2 + 3x$$

$$0 = 3x^2 + 7x - 1$$

$$x = \frac{-7 \pm \sqrt{49 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-7 \pm \sqrt{61}}{6}$$

$$x = \frac{-7 \pm \sqrt{49 + 12}}{6}$$

5.) $6\sin(3x) - \sqrt{27} = 0$

$$\frac{6\sin 3x}{6} = \frac{\sqrt{27}}{6} = \frac{3\sqrt{3}}{6}$$

$$\sin 3x = \frac{3\sqrt{3}}{6}$$

$$\sin 3x = \frac{\sqrt{3}}{2}$$

$$3x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$



$$\frac{3x}{3} = \frac{\pi}{3(3)} + \frac{2\pi n}{3}$$

$$\frac{3x}{3} = \frac{2\pi}{3(3)} + \frac{2\pi n}{3}$$

$$x = \frac{\pi}{9} + \frac{2\pi n}{3}$$

2.) $4\left(\frac{1}{3}\right)^{2x-4} + 5 = 8$

$$4\left(\frac{1}{3}\right)^{2x-4} = 3$$

$$\ln\left(\frac{1}{3}\right)^{2x-4} = \ln\frac{3}{4}$$

$$\frac{2x-4}{\ln\frac{1}{3}} = \frac{\ln\frac{3}{4}}{\ln\frac{1}{3}} + 4$$

$$x = \frac{\frac{\ln\frac{3}{4}}{\ln\frac{1}{3}} + 4}{2}$$

4.) $(x-1)^{\frac{3}{2}} = -8$

$$\sqrt[3]{\sqrt{x-1}} = \sqrt[3]{-8}$$

$$\sqrt{x-1} = -2$$

Not possible
a square root
cannot = a negative

No Solution

$$x = \frac{2\pi}{9} + \frac{2\pi n}{3}$$

$$x = \frac{\ln\frac{3}{4}}{2\ln\frac{1}{3}} + 2$$

6.) Find three different decompositions for the following where $q(k(x)) = h(x)$.

$$h(x) = \frac{5}{(4x+1)^2} - 3$$

$$k(x) = 4x+1$$

$$q(x) = \frac{5}{x^2} - 3$$

$$k(x) = (4x+1)^2$$

$$q(x) = \frac{5}{x} - 3$$

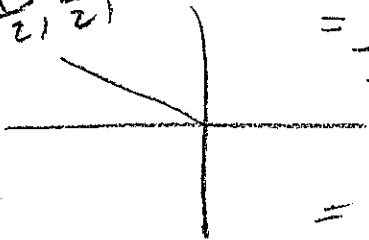
$$k(x) = \frac{5}{(4x+1)^2}$$

$$q(x) = x - 3$$

7.) Evaluate the following. Give only the FUNCTION ANSWER!

a.) $\tan\left(-\frac{7\pi}{6}\right)$

$-\frac{\sqrt{3}}{2}, \frac{1}{2}$



$$= \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$$

$$= \frac{1\sqrt{3}}{-\sqrt{3}\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$$

b.) $\sin^{-1}(-1) = \boxed{-\frac{\pi}{2}}$



c.) $\cos\left(\arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$

$$= \cos\frac{\pi}{4}$$

$$= \boxed{\frac{\sqrt{2}}{2}}$$

8.) Add the following- Find a common denominator.

$$\frac{x}{x+h} - \frac{1}{x+h-x}$$

$$= \frac{x - (x+h)}{x(x+h)}$$

$$= \frac{x - x - h}{x(x+h)}$$

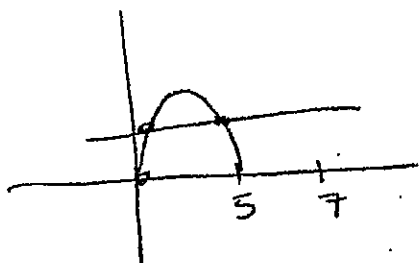
$$= \frac{-h}{x(x+h)} \cdot \frac{1}{1}$$

$$= \boxed{\frac{-1}{x(x+h)}}$$

Name Key Period _____

Mastery #3a.)

- 1.) The volume of a box is given by the function $V(x) = x(10 - 2x)(14 - 2x)$, where x is the height of the box. For what heights will the box have a volume of 25 cubic units?



$$x \approx 4.449 \text{ units}$$

$$x \approx 0.191 \text{ units}$$

- 2.) Solve the equation. Find ALL solutions.

$$4\sin(2x) - \sqrt{8} = 0$$

$$+ \sqrt{8} + \sqrt{8}$$

$$\frac{4\sin(2x)}{4} = \frac{2\sqrt{2}}{4}$$

$$\sin(2x) = \frac{\sqrt{2}}{2}$$

$$2x = \sin^{-1} \frac{\sqrt{2}}{2}$$

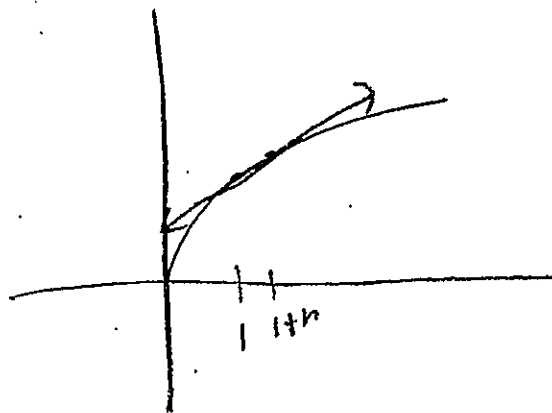
$$\frac{2x}{2} = \frac{\frac{\pi}{4} + \frac{2\pi n}{2}}{2}$$

$$x = \frac{\pi}{8} + \pi n$$

$$2x = \frac{3\pi}{4} + \frac{2\pi n}{2}$$

$x = \frac{3\pi}{8} + \pi n$

- 3.) Sketch a graph of $f(x) = \sqrt{x}$. Find the average rate of change between $x = 1$ and $x = 1 + h$. Find the conjugate to estimate the tangent line slope at $x = 1$.



$$\frac{\sqrt{1+h} - 1}{1+h - 1} = \frac{(\sqrt{1+h} - 1)(\sqrt{1+h} + 1)}{h(\sqrt{1+h} + 1)}$$

secant line

$$= \frac{\sqrt{1+h} - 1}{h(\sqrt{1+h} + 1)} = \frac{1}{\sqrt{1+h} + 1}$$

$h \rightarrow 0$

so tangent line slope at $x = 1$

$$= \frac{1}{1+1} = \frac{1}{2}$$

4.) Sketch a graph of $f(x) = x \sin x$ on the interval $[0, 2\pi]$. Find the following:

Increasing intervals $(0, 2.029)$ $(4.913, 2\pi)$

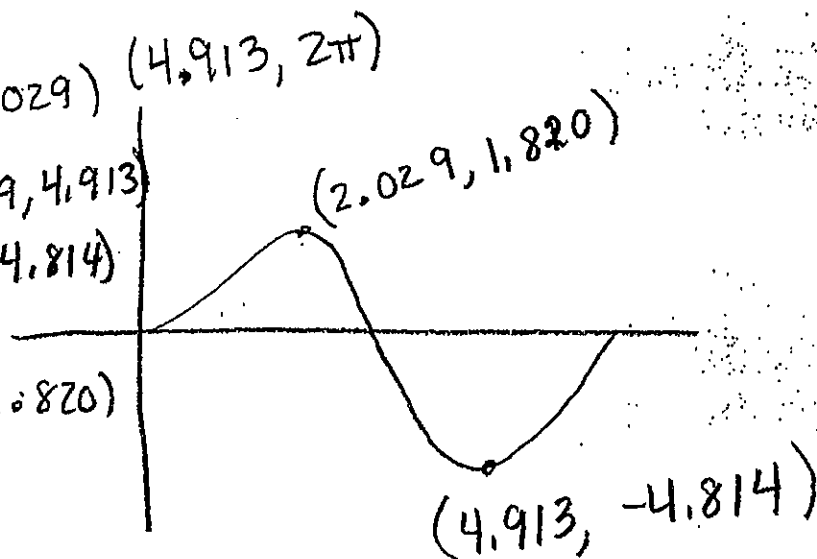
Decreasing intervals $(2.029, 4.913)$

Absolute mins $(4.913, -4.814)$

Relative mins none

Absolute Maxs $(2.029, 1.820)$

Relative maxs none



5.) Solve the equation: $\ln x - \ln(x-1) = 2$

$$e^{\ln\left(\frac{x}{x-1}\right)} = e^2$$

$$\frac{x}{x-1} = e^2(x-1)$$

$$x = e^2 x - e^2$$

$$x - e^2 x = -e^2$$

$$\frac{x(1-e^2)}{1-e^2} = \frac{-e^2}{1-e^2} \approx \boxed{1.157}$$

Mastery 4a.) Formative Assessment

1.) Condense the logarithm and then sketch a graph.

$$f(x) = \ln(2x+2) - \ln 2$$

$$f(x) = \ln\left(\frac{2x+2}{2}\right)$$

$$f(x) = \ln(x+1)$$

$$e^0 = \ln(x+1)$$

$$1 = x+1$$

$$0 = x \quad (0,0)$$

x-intercept $(0,0)$

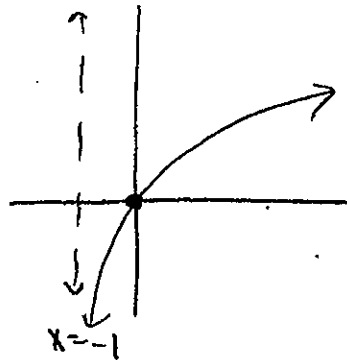
y-intercept $(0,0)$

Domain $\{x | x \in \mathbb{R}, x > -1\}$

Range $\{y | y \in \mathbb{R}\}$

End Behavior in Limit Notation

$$\lim_{x \rightarrow \infty} f(x) = \infty \quad \lim_{x \rightarrow -1^+} f(x) = -\infty$$



$$f(0) = \ln(1) = 0$$

$(0,0)$

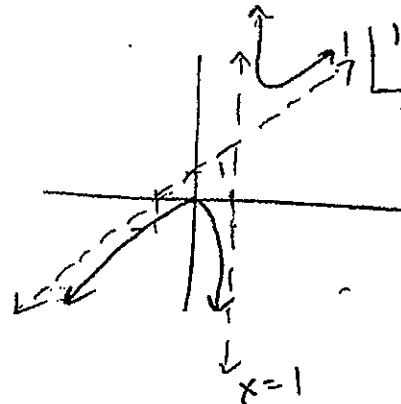
2.) Sketch a graph of $f(x) = \frac{x^2}{x-1}$

End Behavior Asymptote

$$y = x+1$$

x-intercept $(0,0)$

y-intercept $(0,0)$



$$y = x+1 + \frac{1}{x-1}$$

OR

$$x-1 \overline{) x^2 + 0x + 0}$$

$$\underline{-x^2 + x}$$

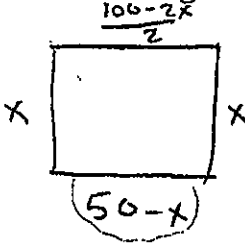
$$x + 0$$

$$\underline{-x + 1}$$

$$1$$

guess 1
0
as $x \rightarrow \infty$

3.) The perimeter of rectangle is 100 feet. Set up a function that models the area of the rectangle. Sketch a graph of your model and find the maximum area possible.



$$A = xy$$

$$A = x(50-x)$$

$$0 = x(50-x)$$

$$x=0 \quad x=50$$

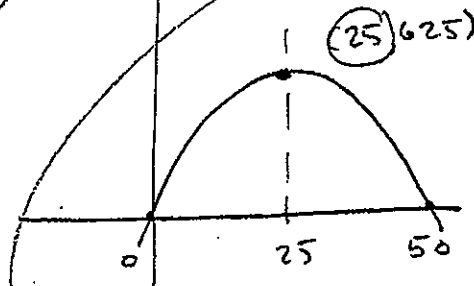
Max Area:
625 ft²

$$100 = 2x + 2y$$

$$-2x$$

$$\frac{100-2x}{2} = \frac{2y}{2}$$

$$50-x = y$$



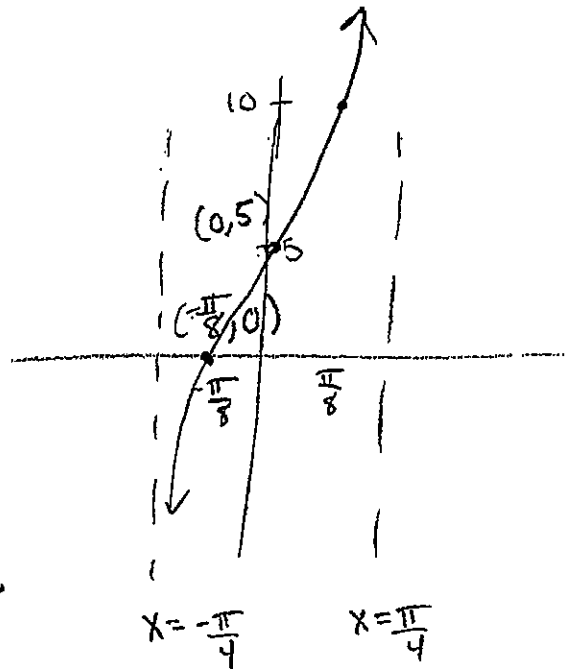
middle $x = 25$

$$A = 25(25) = 625$$

4.) Sketch a graph of the trigonometric function.

$$f(x) = 5 \tan(2x) + 5$$

Transformations | v. stretch of 5 |
 h shrink of $\frac{1}{2}$ | v shift up 5 |
 y-intercept
 $f(0) = 5 \tan(0) + 5 \quad f(0) = 5 \quad (0, 5)$
 x-intercept
 $0 = 5 \tan(2x) + 5 \quad \tan(2x) = -1$
 Domain
 $\{x \mid x \in \mathbb{R}, x \neq -\frac{\pi}{4} + \frac{\pi}{2}n\}$ $2x = -\frac{\pi}{4} + \pi n$
 Range
 $\{y \mid y \in \mathbb{R}\}$
 $x = -\frac{\pi}{8} + \frac{\pi}{2}n$



5.) Verify the identity:

$$\frac{(\sec x - 1)}{(\sec x + 1)} \frac{\sec x + 1}{\tan x} = \frac{\sin x}{1 - \cos x}$$

$$\frac{\sec^2 x - 1}{(\sec x - 1) \tan x} \cdot \frac{\tan^2 x}{(\sec x - 1) (\tan x)} \cdot \frac{\tan x}{\sec x - 1}$$

$$\frac{\tan x}{\sec x - 1} \left(\frac{1}{\sec x - 1} \right)$$

$$\frac{\sin x}{\cos x} \left(\frac{1}{\sec x - 1} \right)$$

$$\frac{\sin x}{1 - \cos x}$$

OR

$$\frac{\sin x (1 + \cos x)}{(1 - \cos x)(1 + \cos x)} = \frac{\sin x (1 + \cos x)}{1 - \cos^2 x}$$

$$= \frac{\sin x (1 + \cos x)}{\sin^2 x} = \frac{1 + \cos x}{\sin x}$$

$$= \frac{1}{\cos x} + \frac{\cos x}{\cos x}$$

$$\frac{\sin x}{\cos x} = \frac{\sec x + 1}{\tan x}$$

Name _____
 Period _____

Average Rate of Change

Use the function $f(x) = x^2$ for the following.

- 1.) Sketch a graph of the function and find the slope of the secant line between $x = -2$ and $x = -1.9$. Sketch the secant line on your graph and write the equation of the secant line.
- 2.) Use $x = -2$ and $x = -2 + h$ to find the slope of the tangent line at $x = -2$. Draw the tangent line on your graph and write the equation of the tangent line.
- 3.) Sketch a graph of the function and find the slope of the secant line between $x = -1$ and $x = -.9$. Sketch the secant line on your graph and write the equation of the secant line.
- 4.) Use $x = -1$ and $x = -1 + h$ to find the slope of the tangent line at $x = -1$. Draw the tangent line on your graph and write the equation of the tangent line.
- 5.) Sketch a graph of the function and find the slope of the secant line between $x = 0$ and $x = 0.1$. Sketch the secant line on your graph and write the equation of the secant line.
- 6.) Use $x = 0$ and $x = 0 + h$ to find the slope of the tangent line at $x = 0$. Draw the tangent line on your graph and write the equation of the tangent line.
- 7.) Sketch a graph of the function and find the slope of the secant line between $x = 1$ and $x = 1.1$. Sketch the secant line on your graph and write the equation of the secant line.
- 8.) Use $x = 1$ and $x = 1 + h$ to find the slope of the tangent line at $x = 1$. Draw the tangent line on your graph and write the equation of the tangent line.
- 9.) What patterns do you notice? What would you guess is the slope of the tangent line at $x = 2$?
- 10.) Find $\frac{f(x+h) - f(x)}{(x+h) - x}$. What did this find?

Use the function $f(x) = 2x^2 - 8$ for the following.

- 11.) Sketch a graph of the function and find the slope of the secant line between $x = 5$ and $x = 5.01$. Sketch the secant line on your graph and write the equation of the secant line.
- 12.) Use $x = 5$ and $x = 5 + h$ to find the slope of the tangent line at $x = 5$. Draw the tangent line on your graph and write the equation of the tangent line.

Use the function $f(x) = -x^2 + 2x$ for the following.

- 13.) Sketch a graph of the function and find the slope of the secant line between $x = -3$ and $x = -3.001$. Sketch the secant line on your graph and write the equation of the secant line.
- 14.) Use $x = -3$ and $x = -3 + h$ to find the slope of the tangent line at $x = -3$. Draw the tangent line on your graph and write the equation of the tangent line.

13.) $m = 8.001$
 $y - 42 = 2002(x - 5)$
 $y - 42 = 20(x - 5)$
 11.) $m = 20.02$
 $m = 20$
 12.) $m = .21$
 9.) The slope is double the x-value
 10.) $2x$

7.) $m = 2.1$
 $y - 1 = 2.1(x - 1)$
 8.) $y - 1 = 2(x - 1)$

Answers
 1.) $m = -3.9$
 $y - 4 = -3.9(x - 2)$
 2.) $m = -4$
 $y - 4 = -4(x - 2)$
 3.) $m = -1.9$
 $y - 1 = -1.9(x - 1)$
 4.) $m = -2$
 $y - 1 = -2(x - 1)$
 5.) $m = -.1$
 $y - 0 = -.1(x - 0)$
 6.) $m = 0$
 $y = 0x$



Rational Functions Worksheet

Sketch a graph of each of the following, giving

- Non-removable discontinuities (vertical asymptotes)
- Removable discontinuities (holes)
- Intercepts
- End Behavior in limit notation/End Behavior asymptotes
- Intercepts
- Domain and Range

ANSWERS ARE ON THE BACK

1. $f(x) = \frac{x^2 - 1}{x + 1}$

2. $f(x) = \frac{x^3 + 2x^2}{x}$

3. $f(x) = \frac{2x^2 - 7x - 15}{x - 5}$

4. $f(x) = \frac{x^3}{x^2 - 4}$

5. $f(x) = \frac{x + 3}{x^2 - 9}$

6. $f(x) = \frac{x^2 + 4x + 4}{x^3 - 4x}$

7. $f(x) = \frac{2x^3 + 2x^2 - 7x + 3}{x - 1}$

8. $f(x) = \frac{x + 1}{x^2 - 4x}$

9. $f(x) = \frac{3x - 1}{3x^2 + 20x - 7}$

10. $f(x) = \frac{x^2 - 7x + 12}{x - 3}$

11. $f(x) = \frac{1}{x^2} - 4$

12. $f(x) = \frac{3x^2 + 20x - 7}{3x - 1}$

13. $f(x) = 2x + \frac{1}{x - 1}$

14. $f(x) = \frac{x^2}{x + 1}$

15. $f(x) = \frac{2x^2 - 8x}{x - 3}$

16. Describe how to find vertical asymptotes. Describe how to find end behavior asymptotes.

17. What is the difference between a vertical asymptote and a horizontal asymptote?

18. What is the difference between a hole and a vertical asymptote?

19. Write AN equation of a rational function that has a removable discontinuity at $x = -2$, a x -intercept of $(5, 0)$, a non-removable discontinuity at $x = 7$, and a horizontal asymptote at $y = 3$.

20. Write AN equation of a rational function with a slant asymptote and a y intercept of $(0, 1)$.

21. Write AN equation of a rational function that has a non-removable discontinuity of $x = 5$ and $x = -1/2$, and has a removable discontinuity at $x = 6$. What is the end behavior of your equation?

22. Write the equation of a rational function with a removable discontinuity at $x = 1$ and a non-removable discontinuity at $x = -4$. What is the end behavior asymptote of your function?

Answers

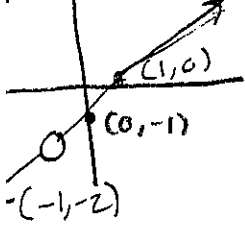
(19) $f(x) = \frac{3(x-5)(x+2)}{(x-7)(x+2)}$

(20) $f(x) = \frac{x^2 + 1}{x + 1}$

(21) $f(x) = \frac{(x-6)}{(x-5)(2x+1)(x-6)}$
 $y = 0$

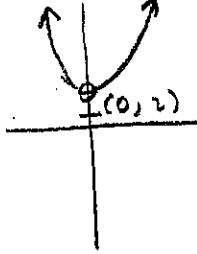
(22) $f(x) = \frac{(x-1)}{(x-1)(x+4)}$ $y = 0$

1) $f(x) = x - 1, x \neq -1$



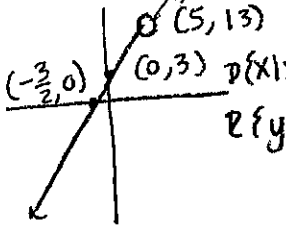
$D \{x | x \in \mathbb{R}, x \neq -1\}$
 $R \{y | y \in \mathbb{R}, y \neq -2\}$

2) $y = x^2 + 2, x \neq 0$



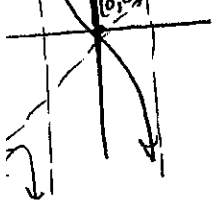
$D \{x | x \in \mathbb{R}, x \neq 0\}$
 $R \{y | y \in \mathbb{R}, y > 2\}$

3) $f(x) = 2x + 3, x \neq 5$



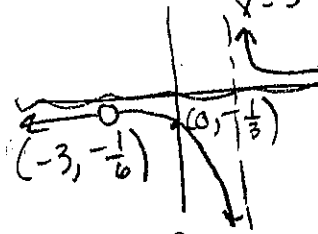
$D \{x | x \in \mathbb{R}, x \neq 5\}$
 $R \{y | y \in \mathbb{R}, y \neq 13\}$

4) $f(x) = x + \frac{4x}{x^2 - 4}$



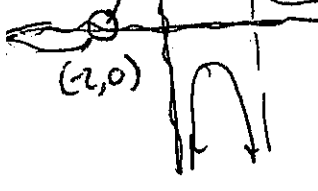
$D \{x | x \in \mathbb{R}, x \neq \pm 2\}$
 $R \{y | y \in \mathbb{R}\}$

5) $f(x) = \frac{1}{x-3}, x \neq 3$



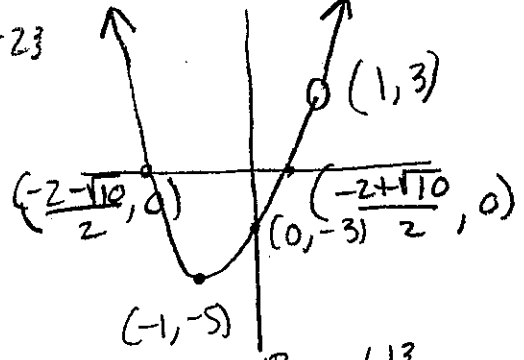
$D \{x | x \in \mathbb{R}, x \neq \pm 3\}$
 $R \{y | y \in \mathbb{R}, y \neq -\frac{1}{6}, y \neq 0\}$

6) $f(x) = \frac{(x+2)}{x(x-2)}, x \neq -2$



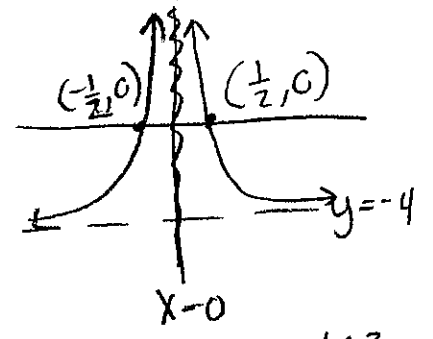
$D \{x | x \in \mathbb{R}, x \neq 0, \pm 2\}$

7) $f(x) = 2x^2 + 4x - 3, x \neq 1$



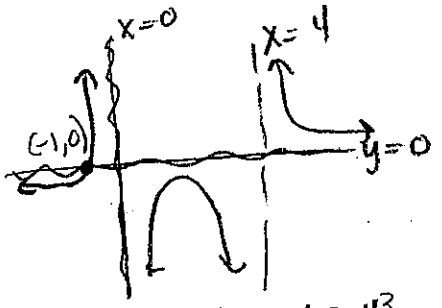
$D \{x | x \in \mathbb{R}, x \neq 1\}$
 $R \{y | y \in \mathbb{R}, y \geq -5\}$

11) $f(x) = \frac{1}{x^2} - 4$



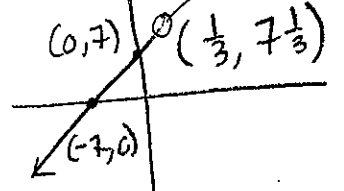
$D \{x | x \in \mathbb{R}, x \neq 0\}$
 $R \{y | y \in \mathbb{R}, y > -4\}$

8)



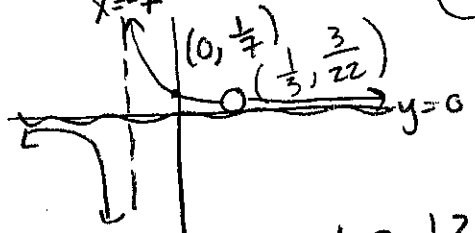
$D \{x | x \in \mathbb{R}, x \neq 0, 4\}$

12) $f(x) = x + 7, x \neq \frac{1}{3}$



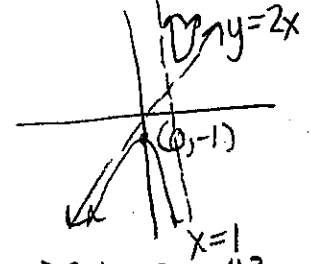
$D \{x | x \in \mathbb{R}, x \neq \frac{1}{3}\}$
 $R \{y | y \in \mathbb{R}, y \neq 7 \frac{1}{3}\}$

9) $f(x) = \frac{1}{x+7}, x \neq \frac{1}{3}$



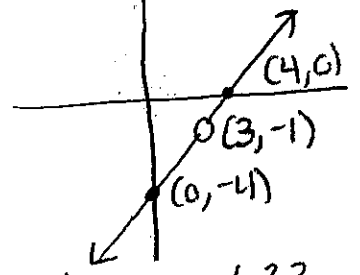
$D \{x | x \in \mathbb{R}, x \neq -7, \frac{1}{3}\}$
 $R \{y | y \in \mathbb{R}, y \neq 0, \frac{3}{22}\}$

13) $f(x) = 2x + \frac{1}{x-1}$



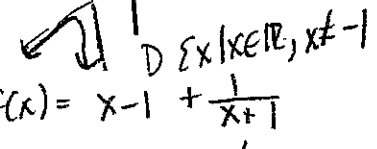
$D \{x | x \in \mathbb{R}, x \neq 1\}$

10) $f(x) = x - 4, x \neq 3$



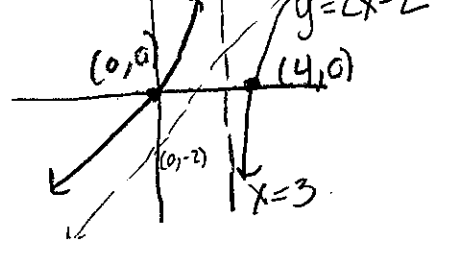
$D \{x | x \in \mathbb{R}, x \neq 3\}$
 $R \{y | y \in \mathbb{R}, y \neq -1\}$

14) $f(x) = x - 1 + \frac{1}{x+1}$



$D \{x | x \in \mathbb{R}, x \neq -1\}$

15) $f(x) = 2x - 2 - \frac{6}{x-3}$



Name _____

Period _____

Precalculus Review #1

- 1.) Find the average rate of change for $f(x) = \log_2 x$ between $[4, 16]$.
2.) Solve the following equations.

a.) $7x^3 - 12x^2 - 4x = 0$

b.) $4 \ln x - \ln 2 = 8$

c.) $\frac{1}{3} \cdot 2^{3x} - 4 = 7$

d.) $\frac{1}{x+4} - 2 = 8$

e.) $\cos 2x = 1$

- 3.) Sketch a graph of each of the following. *Give the end behavior in limit notation.*

a.) $f(x) = \begin{cases} x^2 - 1 & x > 0 \\ 3x & x \leq 0 \end{cases}$

b.) $f(x) = \frac{2x+2}{x^2-1}$

c.) $f(x) = x^3 + 2x - 3$

d.) $f(x) = -\ln x + 1$

1.) $\frac{f(16) - f(4)}{16 - 4}$
 $\frac{\log_2 16 - \log_2 4}{12}$
 $= \frac{4 - 2}{12}$
 $= \frac{2}{12} = \frac{1}{6}$

2 d.) $\frac{1}{x+4} - 2 = 8$
 $\frac{1(x+4)}{x+4} = 10(x+4)$
 $1 = 10x + 40$
 $-40 = 10x - 40$
 $\frac{-39}{10} = \frac{10x}{10}$
 $x = -3.9$

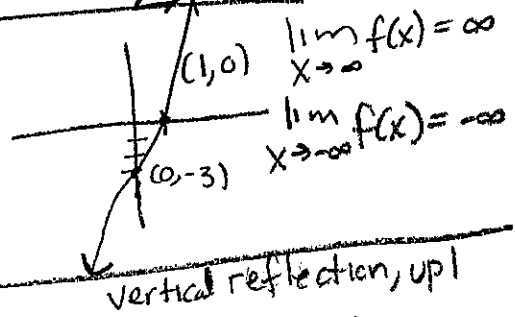
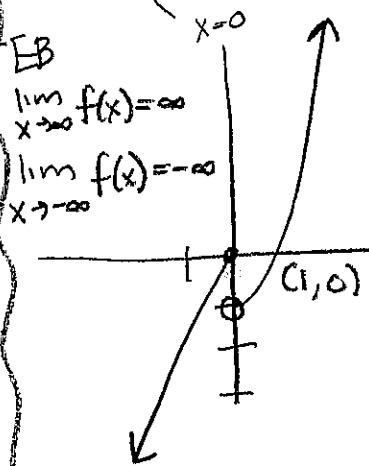
3 c.) $f(x) = x^3 + 2x - 3$
 $\begin{array}{r|rrrr} 1 & 1 & 0 & 2 & -3 \\ & & 1 & 3 & 0 \\ \hline & 1 & 1 & 3 & 0 \end{array} (1,0)$
 $x^2 + x + 3 = 0$
 $x = \frac{-1 \pm \sqrt{1 - 4(1)(3)}}{2}$
 $x = \frac{-1 \pm \sqrt{-11}}{2}$

2 a.) $7x^3 - 12x^2 - 4x = 0$
 $x(7x^2 - 12x - 4) = 0$
 $(7x + 2)(x - 2) = 0$
 $x = -\frac{2}{7}, x = 2$

3 a.) $f(x) = \begin{cases} x^2 - 1 & x > 0 \\ 3x & x \leq 0 \end{cases}$

$x = \frac{-1 \pm i\sqrt{11}}{2}$ Imaginary

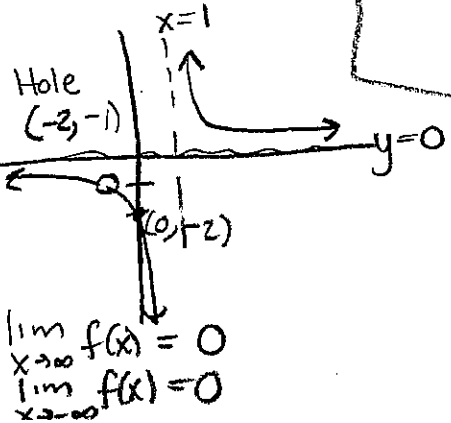
3.) $4 \ln x - \ln 2 = 8$
 $\ln \frac{x^4}{2} = 8$
 $\frac{x^4}{2} = e^8$
 $x^4 = 2e^8$
 $x = \sqrt[4]{2e^8} \approx 8.787$



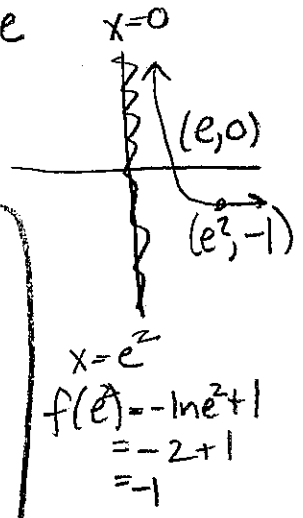
~~$x = \sqrt[4]{2e^8}$~~ You cannot have a log of a negative #

3 d.) $f(x) = -\ln x + 1$
 $x \text{ int } 0 = -\ln x + 1$
 $-1 = -\ln x$
 $1 = \ln x$
 $x = e$

b) $f(x) = \frac{2x+2}{x^2-1}$
 $f(x) = \frac{2(x+1)}{(x-1)(x+1)} \quad x \neq -1$



3.) $\frac{1}{3} \cdot 2^{3x} - 4 = 7$
 $\frac{1}{3} \cdot 2^{3x} = 11 \cdot 3$
 $\ln 2^{3x} = \ln 33$
 $\frac{3x(\ln 2)}{3 \ln 2} = \frac{\ln 33}{3 \ln 2}$
 $x = \frac{\ln 33}{3 \ln 2} \approx 1.681$



Precalculus Review #2

- 1.) Sketch a graph of $f(x) = \begin{cases} 2x+4 & x \geq 0 \\ 5 & x < 0 \end{cases}$. Label all intercepts, the domain and the range.
- 2.) Find the end behavior in limit notation of $f(x) = -5\left(\frac{1}{2}\right)^x - 4$.
- 3.) Solve the equation: $2x^4 - x^3 + x^2 - x - 1 = 0$
- 4.) Rewrite the following as a natural logarithm. $\log_2 7$
- 5.) Evaluate each of the following if possible:
 - a.) $\ln e^2$
 - b.) $\ln 1$
 - c.) $\ln 0$

Name _____

Precalculus Review #3

Solve each equation. Give **all** solutions and simplify your solutions.

1. $2x^3 + 3x^2 - 8x = -3$

2. $3^{x-1} + 4 = 8$

3. $\frac{1}{x+1} - \frac{2}{x-4} = \frac{5}{x^2 - 3x - 4}$

4. $\log_2 \frac{1}{2} + \log_2(x-1) = 2$

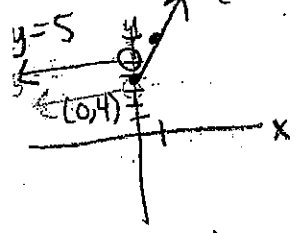
5. $-4 \cot\left(\frac{x}{3}\right) + 4\sqrt{3} = 0$

6. Long divide the following:

$$\begin{array}{r} x^3 - 5x^2 + 8x - 3 \\ \underline{x^2 + 1} \end{array}$$

Key

1.) $f(x) = \begin{cases} 2x+4 & x \geq 0 \\ 5 & x < 0 \end{cases}$



$y_{int} (0, 4)$
 x_{int} none
 $D \in \mathbb{R} \mid x \in \mathbb{R}$
 $R \ni y \mid y \in \mathbb{R}, y \geq 4$

3.) $2x^4 - x^3 + x^2 - x - 1 = 0$

$x=1$ $\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ & 2 & 1 & 2 & 1 \\ \hline 2 & 1 & 2 & 1 & 0 \end{array}$

$2x^3 + x^2 + 2x + 1 = 0$
 $x^2(2x+1) + 1(2x+1) = 0$
 $(x^2+1)(2x+1) = 0$
 $x^2+1=0 \quad 2x+1=0$
 $-1 \quad -1 \quad -1 \quad -1$

$\sqrt{x^2+1}$
 $x = \pm i$
 $\frac{2x}{2} = \frac{-1}{2}$
 $x = -\frac{1}{2}$

5a) $= 2 \ln e = 2$

$\ln e^2 = 2$

b) $\ln 1 = 0$

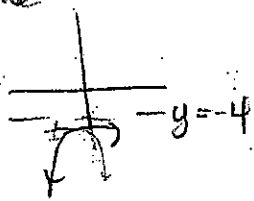
$e^? = 1 \rightarrow$

c) $\ln 0$

$e^? = 0$ ← Not possible

2.) $f(x) = -5\left(\frac{1}{2}\right)^x - 4$

$\frac{1}{2}$ flip stretch of 5 down 4
 $\frac{1}{2}$ none



$\lim_{x \rightarrow \infty} f(x) = -4$

$\lim_{x \rightarrow -\infty} f(x) = -\infty$

4) $\log_2 7 = \frac{\ln 7}{\ln 2}$

Key

1.) $2x^3 + 3x^2 - 8x + 3 = 0$

$\begin{array}{r|rrrr} 1 & 2 & 3 & -8 & 3 \\ & & 2 & 5 & -3 \\ \hline & 2 & 5 & -3 & 0 \end{array}$

$2x^2 + 5x - 3$

$(2x-1)(x+3) = 0$

$x = \frac{1}{2}, x = -3, x = 1$

3.) $\left(\frac{1}{x+1} - \frac{2}{x-4}\right) = \frac{5}{(x-4)(x+1)}$

$1(x-4) - 2(x+1) = 5$
 $x \neq -1$
 $x \neq 4$

$x-4 - 2x-2 = 5$
 $-x-6 = 5$
 $+6 \quad +6$

$-x = 11$
 $\frac{-1x}{-1} = \frac{11}{-1}$

$x = -11$

5.) $-4 \cot\left(\frac{x}{3}\right) + 4\sqrt{3} = 0$

$\frac{-4 \cot\left(\frac{x}{3}\right)}{-4} = \frac{-4\sqrt{3}}{-4}$

$\cot\left(\frac{x}{3}\right) = \sqrt{3}$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{x}{3} = \frac{\pi}{6} + \pi n$

$\frac{x}{3} = \frac{\pi}{6} + \pi n$

$x = \frac{\pi}{2} + 3\pi n$

2.) $3^{x-1} + 4 = 8$
 $-4 \quad -4$

$\ln 3^{x-1} = \ln 4$

$x-1 (\ln 3) = \ln 4$

$x-1 = \frac{\ln 4}{\ln 3} + 1$

$x = \frac{\ln 4}{\ln 3} + 1$

4.) $\log_2 \frac{1}{2} + \log_2 (x-1) = 2$

$-1 + \log_2 (x-1) = 2$
 $+1 \quad +1$

$\log_2 (x-1) = 3$
 $2 \quad 2$

$x-1 = 8$
 $+1 \quad +1$

$x = 9$

6)

$x^2 + 1 \mid \begin{array}{r} x^3 - 5x^2 + 8x - 3 \\ -x^3 \\ \hline -5x^2 + 8x - 3 \\ +5x^2 \\ \hline 8x - 3 \\ +5 \\ \hline 8x + 2 \end{array}$

$$\frac{(1-x)\epsilon}{1} = \frac{1+x\epsilon}{1} =$$

$$\frac{x-1}{1} \frac{b}{1} = \frac{b}{x} - \frac{b}{1} \quad \textcircled{\epsilon}$$

Precalculus Review #4

Simplify the following expressions so that you can graph them. Then graph the expressions.

1.) $\frac{x^{\frac{1}{2}} + x}{\sqrt{x}}$

2.) $\frac{x}{x-1} - \frac{1}{x}$

3.) $\frac{\sin^2 x}{1 - \cos x}$

4.) $\frac{-\sqrt{20}(x-1)^{\frac{1}{3}}}{\sqrt{45}}$

5.) $\ln x - \ln e^4$

6.) $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$

7.) $\sqrt{\frac{1}{9} - \frac{x}{9}}$

$$\frac{(1-x)\epsilon}{1} = \frac{1+x\epsilon}{1} =$$

$$\frac{x-1}{1} \frac{b}{1} = \frac{b}{x} - \frac{b}{1} \quad \textcircled{\epsilon}$$

Precalculus Review #4

Simplify the following expressions so that you can graph them. Then graph the expressions.

1.) $\frac{x^{\frac{1}{2}} + x}{\sqrt{x}}$

2.) $\frac{x}{x-1} - \frac{1}{x}$

3.) $\frac{\sin^2 x}{1 - \cos x}$

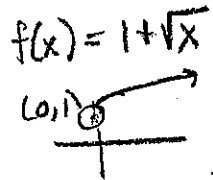
4.) $\frac{-\sqrt{20}(x-1)^{\frac{1}{3}}}{\sqrt{45}}$

5.) $\ln x - \ln e^4$

6.) $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$

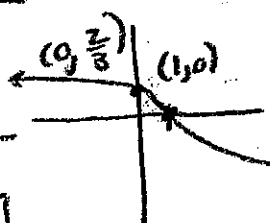
7.) $\sqrt{\frac{1}{9} - \frac{x}{9}}$

1.) $\frac{\sqrt{x}(1+\sqrt{x})}{\sqrt{x}}$

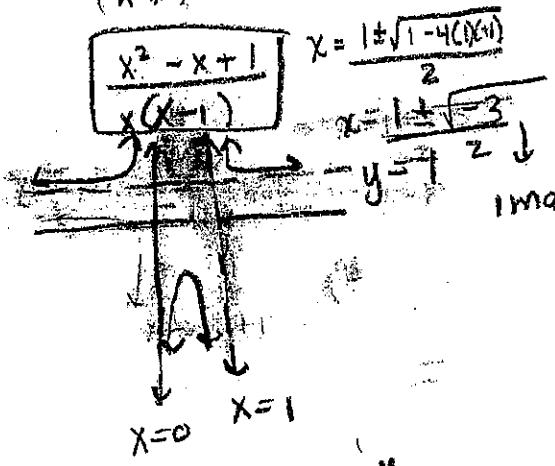


removable discontinuity (0, 1)

4.) $\frac{-\sqrt[3]{20}\sqrt{x-1}}{\sqrt{45}}$
 $\frac{-2\sqrt[3]{5}\sqrt{x-1}}{3\sqrt{5}}$
 $\frac{-2\sqrt[3]{5}\sqrt{x-1}}{3\sqrt{5}}$

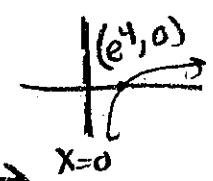


2.) $\frac{x(x)}{(x-1)(x)} - \frac{1(x-1)}{x(x-1)}$



imaginary

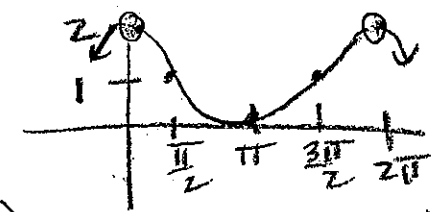
5.) $\ln x - \ln e^4$
 $= \ln x - 4$



$0 = \ln x - 4$
 $4 = \ln x$
 $x = e^4$

3.) $\frac{\sin^2 x}{1 - \cos x} = \frac{1 - \cos^2 x}{1 - \cos x}$

$= \frac{(1 - \cos x)(1 + \cos x)}{1 - \cos x}$
 $= 1 + \cos x$

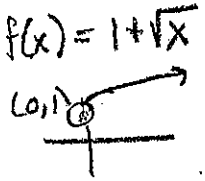


6.) $\frac{(\cos x)\cos x}{(\cos x)(1 + \sin x)} + \frac{(1 + \sin x)(1 + \sin x)}{\cos x(1 + \sin x)}$
 $\frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1 + \sin x)}$

$= \frac{2 + 2\sin x}{\cos x(1 + \sin x)} = \frac{2(1 + \sin x)}{\cos x(1 + \sin x)}$
 $= 2 \sec x$

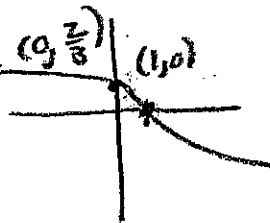


1.) $\frac{\sqrt{x}(1+\sqrt{x})}{\sqrt{x}}$

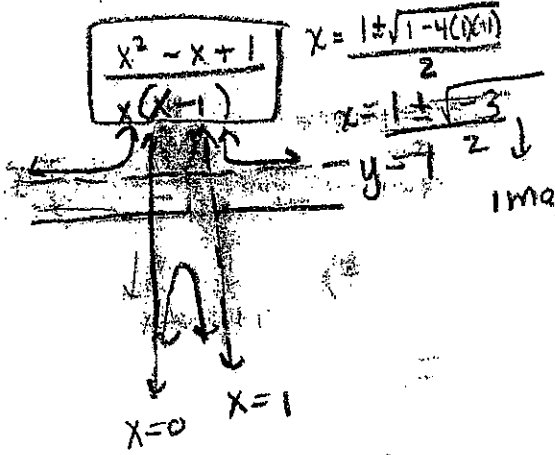


removable discontinuity (0, 1)

4.) $\frac{-\sqrt[3]{20}\sqrt{x-1}}{\sqrt{45}}$
 $\frac{-2\sqrt[3]{5}\sqrt{x-1}}{3\sqrt{5}}$
 $\frac{-2\sqrt[3]{5}\sqrt{x-1}}{3\sqrt{5}}$

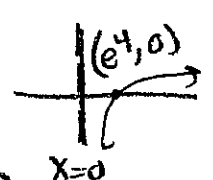


2.) $\frac{x(x)}{(x-1)(x)} - \frac{1(x-1)}{x(x-1)}$



imaginary

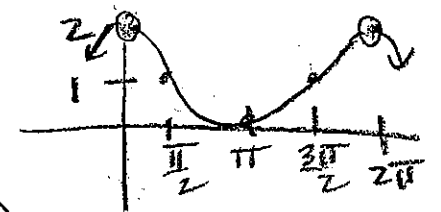
5.) $\ln x - \ln e^4$
 $= \ln x - 4$



$0 = \ln x - 4$
 $4 = \ln x$
 $x = e^4$

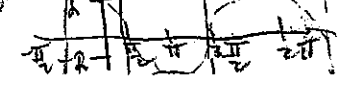
3.) $\frac{\sin^2 x}{1 - \cos x} = \frac{1 - \cos^2 x}{1 - \cos x}$

$= \frac{(1 - \cos x)(1 + \cos x)}{1 - \cos x}$
 $= 1 + \cos x$



6.) $\frac{(\cos x)\cos x}{(\cos x)(1 + \sin x)} + \frac{(1 + \sin x)(1 + \sin x)}{\cos x(1 + \sin x)}$
 $\frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1 + \sin x)}$

$= \frac{2 + 2\sin x}{\cos x(1 + \sin x)} = \frac{2(1 + \sin x)}{\cos x(1 + \sin x)}$
 $= 2 \sec x$



Review Guide for College Algebra & Trigonometry

- I) **Functional notation.** Given that $f(x) = 3x^2 - 2x + 5$ and $g(x) = 3x - 1$, find each of the following:

$$1) \quad g(4) - f(-2) = (3(4) - 1) - (3(-2)^2 - 2(-2) + 5) \\ = (12 - 1) - (12 - 4 + 5) = 11 - (12 + 9) = 11 - 21 = -10$$

$$2) \quad g(x+h) - g(x) = (3(x+h) - 1) - (3x - 1) = 3x + 3h - 1 - 3x + 1 = 3h$$

$$3) \quad (g \circ f)(x) = g(f(x)) = 3(3x^2 - 2x + 5) - 1 = 9x^2 - 6x + 15 - 1 = 9x^2 - 6x + 14$$

$$4) \quad g^{-1}(x) \quad \begin{matrix} x+1 = 3y-1 \\ +1 & +1 \end{matrix} \quad \frac{x+1}{3} = \frac{3y}{3} \quad \boxed{g^{-1}(x) = \frac{x+1}{3}}$$

- II) **Linear inequalities in one variable.** Solve. Graph on a number line and write in interval notation.

$$1) \quad 3(x-4) - (x+1) \leq x-12$$

$$3x - 12 - x - 1 \leq x - 12$$

$$2x - 13 \leq x - 12$$

$$x - 13 \leq -12$$

$$x \leq 1$$

Graph: $x \leq 1$ on a number line from 0 to 1. Interval notation: $(-\infty, 1]$

- III) **Exponents and polynomials.** Simplify. Write all answers with positive exponents.

$$1) \quad (3x-5)^4 = (3x-5)(3x-5)(3x-5)(3x-5) = (9x^2 - 30x + 25)(9x^2 - 30x + 25) \\ = 81x^4 - 270x^3 + 225x^2 - 270x^3 + 900x^2 - 750x + 225x^2 - 750x + 625 \\ = 81x^4 - 540x^3 + 1350x^2 - 1500x + 625$$

$$2) \quad (4x^2y^6z)^2 (-x^5y^7z^8)^6 = 16x^4y^{12}z^2 \cdot x^{30}y^{42}z^{48} = 16x^{34}y^{54}z^{50}$$

$$3) \quad \frac{4x^4 - 4x^3 + 3x^2 - 5x + 2}{2x - 3}$$

- IV) **Complex Numbers.** Simplify.

$$1) \quad \frac{4x^4 - 4x^3 + 3x^2 - 5x + 2}{2x - 3} = 2x^3 + x^2 + 3x + 2 + \frac{8}{2x - 3}$$

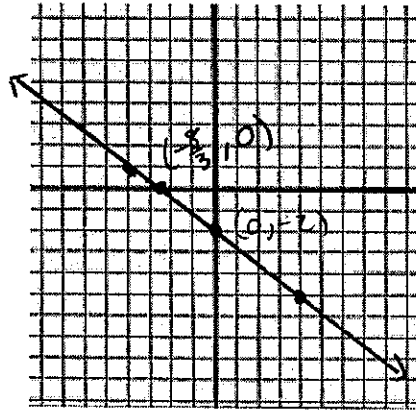
$$2) \quad \sqrt{-25} \sqrt{-81} = 5i \cdot 9i = 45i^2 = 45(-1) = -45$$

$$\frac{(3-4i)(6-2i)}{(6+2i)(6-2i)} = \frac{18 - 6i - 24i + 8i^2}{36 + 12i - 12i - 4i^2} = \frac{18 - 30i - 8}{36 - 4(-1)} = \frac{10 - 30i}{40} = \frac{1}{4} - \frac{3}{4}i$$

$$3) \quad (5+2i)(-3-5i) = -15 - 25i - 6i - 10i^2 = -15 - 31i + 10 = -5 - 31i$$

V) Equations and inequalities in two variables. Graph on a coordinate system. Identify the intercepts and the slope of the linear equation.

1) $y = (-3/4)x - 2$



$$\begin{aligned}
 z &= -1 \\
 x + 2y - z &= 9 \\
 x + y + z &= 8 \\
 &\downarrow \\
 x + 2y &= 11 \\
 x + y &= 7 \\
 &\downarrow \\
 x + 2y &= 11 \\
 -x - y &= -7 \\
 \hline
 y &= 4 \\
 x &= 3
 \end{aligned}$$

2) Solve the system:

- A. $2x - 2y - z = -1$
- B. $x + 2y + 2z = 9$
- C. $x + y - z = 8$

$(-C+B)$
 $(-2C+A)$

$$\begin{aligned}
 2x - 2y - z &= -1 \\
 x + 2y + 2z &= 9 \\
 -x + y + z &= -8
 \end{aligned}$$

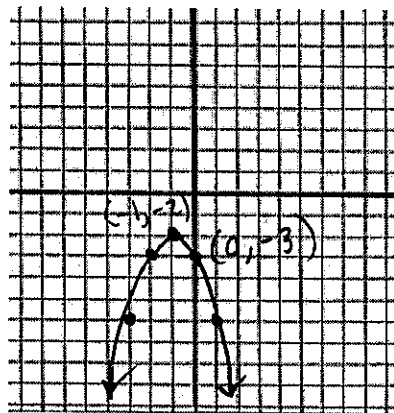
$$\begin{aligned}
 -4y + z &= -17 \rightarrow \\
 y + 3z &= 1
 \end{aligned}$$

$$\begin{aligned}
 -4y + z &= -17 \\
 4y + 12z &= 4 \\
 \hline
 13z &= -13
 \end{aligned}$$

$x = 3 \quad y = 4 \quad z = -1$

VI) Quadratic equations and functions. Graph on a coordinate system. Identify the coordinates of the vertex and the intercepts.

1) $g(x) = -(x + 1)^2 - 2$



VII) Rational functions and expressions.

1) Find the domain of

$$f(a) = \frac{a^2 + 2a - 3}{3a^2 + 11a + 6}$$

$$\left\{ x \mid x \in \mathbb{R}, x \neq -\frac{2}{3}, x \neq -3 \right\}$$

$$f(a) = \frac{(a+3)(a-1)}{(3a+2)(a+3)}$$

$$\frac{3p}{(p+2)(p+3)} = \frac{5p}{(p+3)(p-1)} - \frac{2}{(p+2)(p-1)}$$

$$3p(p-1) = 5p(p+2) - 2(p+3)$$

$$3p^2 - 3p = 5p^2 + 10p - 2p - 6$$

$$3p^2 - 3p = 5p^2 + 8p - 6$$

$$\frac{3p}{p^2 + 5p + 6} = \frac{5p}{p^2 + 2p - 3} - \frac{2}{p^2 + p - 2}$$

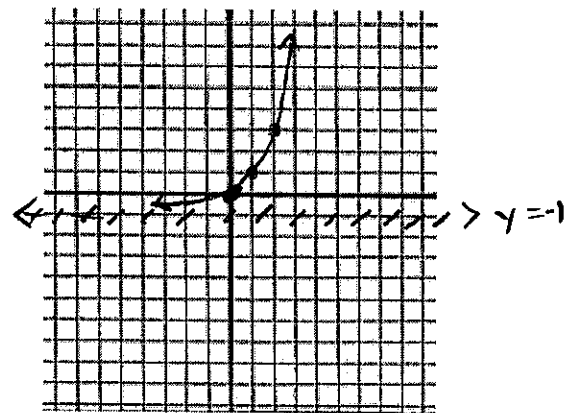
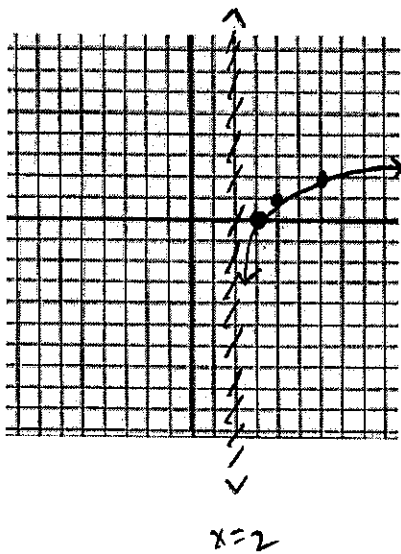
LCD: $(p-1)(p+2)(p+3)$
 $3p^2 - 3p = 5p^2 + 8p - 6$
 $0 = 2p^2 + 11p - 6$
 $0 = (2p-1)(p+6)$
 $0 = 2p-1 \quad p+6=0$
 $p = \frac{1}{2} \quad p = -6$

VIII) Logarithms and exponentials.

- 1) Condense into a single log and simplify: $\log(10) - \log(5)$ $\log\left(\frac{10}{5}\right)$
 $\log 2$
- 2) Expand into sums, differences, and products.
 $\log\left(\frac{abc^3}{d^2}\right) = \log(abc^3) - \log(d^2)$
 $\log a + \log b + \log c^3 - \log d^2$
- 3) Solve: $\log a + \log b + 3 \log c - 2 \log d$
 a) $\ln x + \ln(2x+1) = 0$
 $\ln(2x^2+x) = 0 \quad 2x^2+x=1$
 $2x=3 \quad 2x^2+x-1=0$
 $(2x-1)(x+1)=0$
 $x=\frac{1}{2} \quad x \neq -1$
 $x = \frac{3}{2}$
- 4) Graph: $3^{2x} = 3^3$

a) $g(x) = \log(x-2)$

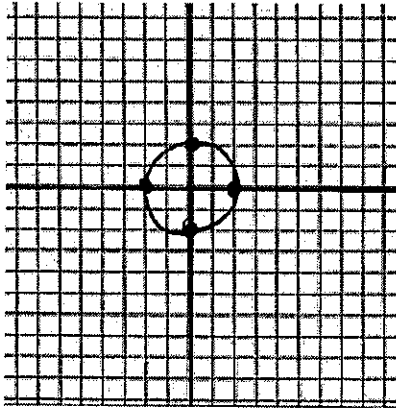
b) $p(x) = 2^x - 1$





IX) Miscellaneous graphing. Graph on a coordinate system.

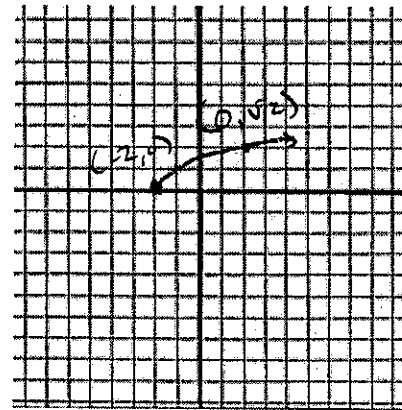
1) $x^2 + y^2 = 4$



center
(0,0)
x ints:
(-2,0)
(2,0)
y ints:
(0,2)
(0,-2)

2)

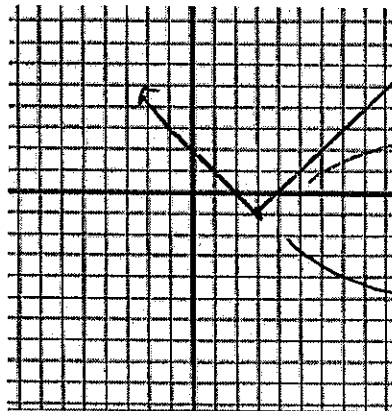
$f(x) = \sqrt{x+2}$ for $x \geq -2$



x int:
(-2,0)

y int:
(0, sqrt(2))

3) $f(x) = |x-3| - 1$



y int:
(0,2)

x-ints: (2,0) (4,0)

(3,-1)

X) Polynomial functions.

1) Find all zeros of the function: $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

XI) Matrices and determinants.

1) Evaluate the determinant of

$$\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}$$

$(3)(5) - (2)(4)$

$15 - 8$

$\boxed{7}$

$$\begin{array}{r} 1 \quad -2 \quad 13 \quad -21 \quad 2 \quad 8 \\ \hline \quad -2 \quad 11 \quad -10 \quad -8 \quad 0 \\ 2 \quad -2 \quad 11 \quad -10 \quad -8 \quad 0 \\ \hline \quad -4 \quad 14 \quad 8 \quad 0 \\ \hline -2 \quad 7 \quad 14 \quad 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-1)(x-2)(-2x^2+7x+4) \\ &= (x-1)(x-2)(2x^2-7x-4) \\ &= -(x-1)(x-2)(2x+1)(x-4) \end{aligned}$$

$x=1 \quad x=2 \quad x=-\frac{1}{2} \quad x=4$



XII) Sequences and series.

- 1) Write the first five terms of the arithmetic sequence

$$A_1 = 6, A_{k+1} = A_k - 5 \quad 6, 1, -4, -9, -14$$

- 2) Find the n^{th} partial sum of the arithmetic sequence

$$8, 20, 32, 44, \dots \quad n = 10 \quad \sum_{n=1}^{10} (12n-4) = \frac{(8 + 120-4)10}{2} = \frac{1240}{2} = 620$$

$$a_n = 12n - 4$$

- 3) Write the first five terms of the geometric sequence $a_1 = 1, r = 1/3$

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}$$

- 3) Find the given sum. Round your answer to the nearest tenth.

$$\sum_{k=1}^{10} 8 \left(\frac{-1}{4} \right)^{k-1} = \frac{8(1 - (-\frac{1}{4})^{10})}{1 + \frac{1}{4}} \approx 6.4$$

XIII) Trigonometry.

- 1) Find the indicated values.

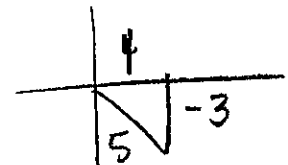
a) $\sin 30^\circ = \sin \frac{\pi}{6} = \frac{1}{2}$

b) $\cos 45^\circ = \frac{\sqrt{2}}{2}$

c) $60^\circ = \frac{\pi}{3}$ radians

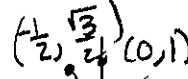
d) π radians = 180°

e) in quadrant IV, if $\cos x = 4/5$, then $\sin x = -\frac{3}{5}$



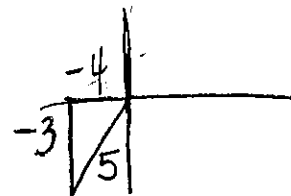
f) in quadrant III, if $\tan x = 3/4$, then $\cos x = -\frac{4}{5}$

g) $\sin(2\pi/3) = \frac{\sqrt{3}}{2}$

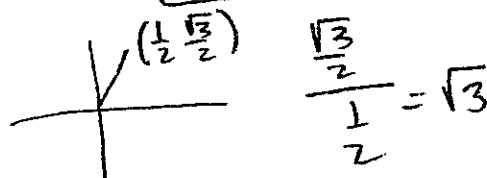


h) $\cos(\pi/2) = 0$

i) if $\tan x = 5/6$, then $\cot x = \frac{6}{5}$



j) $\tan(\pi/3) = \sqrt{3}$



Review Guide for College Algebra & Trigonometry

I) **Functional notation.** Given that $f(x) = 3x^2 - 2x + 5$ and $g(x) = 3x - 1$, find each of the following:

1) $g(4) - f(-2)$

2) $g(x + h) - g(x)$

3) $(g \circ f)(x)$

4) $g^{-1}(x)$

II) **Linear inequalities in one variable.** Solve. Graph on a number line and write in interval notation.

1) $3(x - 4) - (x + 1) \leq x - 12$

III) **Exponents and polynomials.** Simplify. Write all answers with positive exponents.

1) $(3x - 5)^4$

2) $(4x^2y^6z)^2 (-x^5y^7z^8)^6$

3)

$$\frac{4x^4 - 4x^3 + 3x^2 - 5x + 2}{2x - 3}$$

IV) **Complex Numbers.** Simplify.

1)

$$\sqrt{-25} \sqrt{-81}$$

2)

$$\frac{3-4i}{6+2i}$$

3)

$$(5+2i)(-3-5i)$$

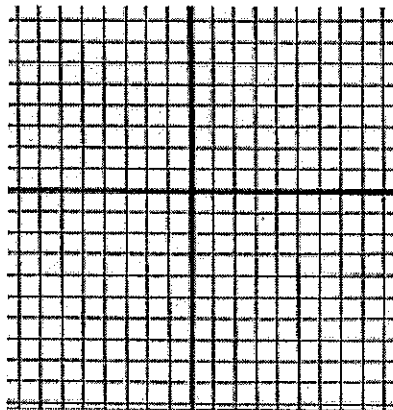
key I) 1) -10 2) 3h 3) $9x^2 - 6x + 14$ 4) $\frac{x+1}{3}$ II) $x \leq 1$ $(-\infty, 1]$

III) 1) $81x^4 - 540x^3 + 1350x^2 - 1500x + 625$ 2) $4x^{34}y^{54}z^{50}$ 3) $2x^3 + x^2 + 3x + 2 + \frac{8}{2x-3}$

IV) 1) -45 2) $\frac{1}{4} - \frac{3}{4}i$ 3) $-5 - 31i$

V) **Equations and inequalities in two variables.** Graph on a coordinate system. Identify the intercepts and the slope of the linear equation.

1) $y = (-3/4)x - 2$



2) Solve the system:

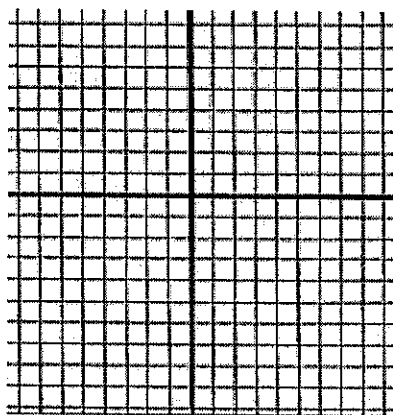
$$2x - 2y - z = -1$$

$$x + 2y + 2z = 9$$

$$x + y - z = 8$$

VI) **Quadratic equations and functions.** Graph on a coordinate system. Identify the coordinates of the vertex and the intercepts.

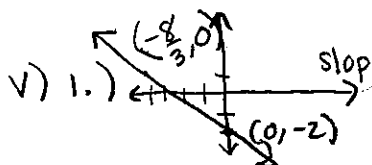
1) $g(x) = -(x + 1)^2 - 2$



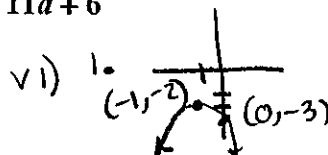
VII) **Rational functions and expressions.**

1) Find the domain of

$$f(a) = \frac{a^2 + 2a - 3}{3a^2 + 11a + 6}$$



2) $(3, 4, -1)$



VII) $\{a \mid a \in \mathbb{R}, a \neq -\frac{2}{3}, a \neq -3\}$

2) Solve:

$$\frac{3p}{p^2 + 5p + 6} = \frac{5p}{p^2 + 2p - 3} - \frac{2}{p^2 + p - 2}$$

VIII) Logarithms and exponentials.

1) Condense into a single log and simplify: $\log(10) - \log(5)$

2) Expand into sums, differences, and products.

$$\log\left(\frac{abc^3}{d^2}\right)$$

3) Solve:

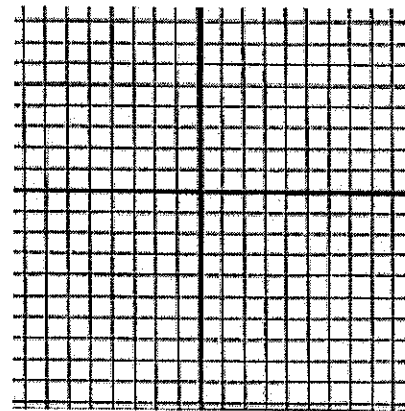
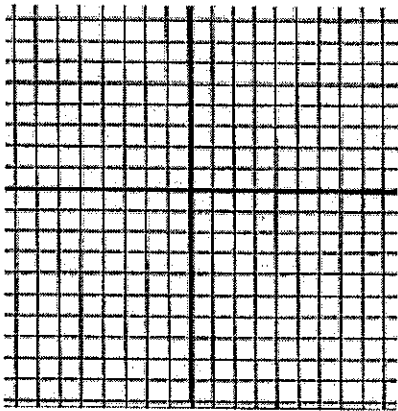
a) $\ln x + \ln(2x + 1) = 0$

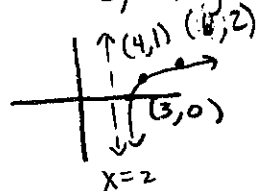
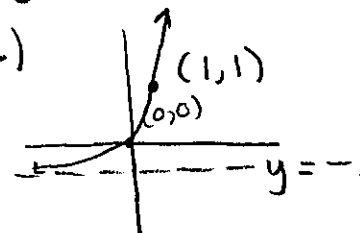
b) $9^x = 27$

4) Graph:

a) $g(x) = \log_2(x - 2)$

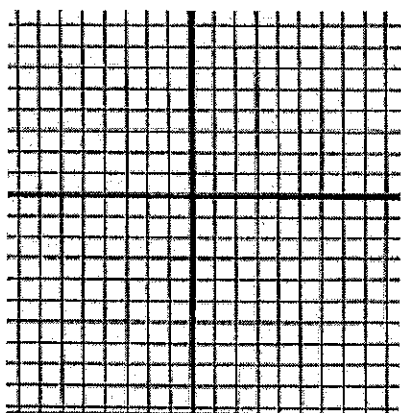
b) $p(x) = 2^x - 1$



VII) 2.) $p = \frac{1}{2}$ or $p = -6$
 3.) a) $x = \frac{1}{2}$ b) $x = \frac{3}{2}$
 VIII) 1) $\log 2$ 2) $\log a + \log b + 3 \log c - 2 \log d$
 4.) 
 5.) 

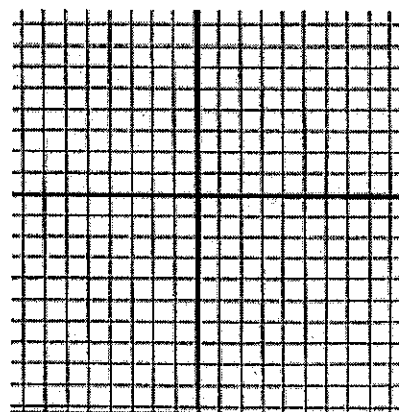
IX) Miscellaneous graphing. Graph on a coordinate system.

1) $x^2 + y^2 = 4$

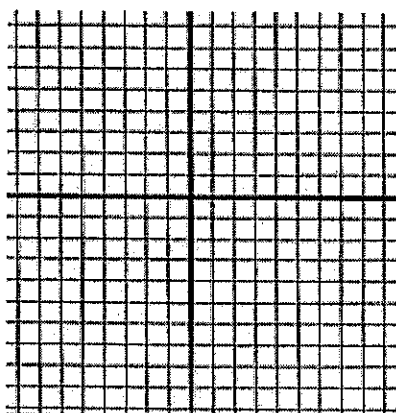


2)

$f(x) = \sqrt{x+2}$ for $x \geq -2$



3) $f(x) = |x - 3| - 1$



X) Polynomial functions.

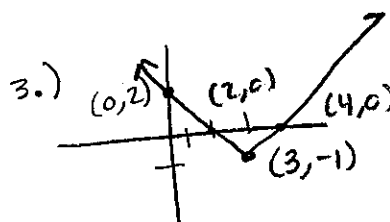
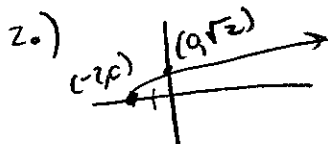
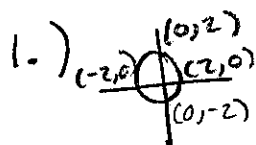
1) Find all zeros of the function: $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

~~XI) Matrices and determinants.~~

1) Evaluate the determinant of

$$\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}$$

IX)



x) $x = 1, x = 2, x = -\frac{1}{2}, x = 4$

~~XII~~ Sequences and series.

- 1) Write the first five terms of the arithmetic sequence

$$A_1 = 6, A_{k+1} = A_k - 5$$

- 2) Find the n^{th} partial sum of the arithmetic sequence

$$8, 20, 32, 44, \dots \quad n = 10$$

- 3) Write the first five terms of the geometric sequence $a_1 = 1, r = 1/3$

- 3) Find the given sum. Round your answer to the nearest tenth.

$$\sum_{k=1}^{10} 8 \left(\frac{-1}{4} \right)^{k-1}$$

XIII) Trigonometry.

- 1) Find the indicated values.

a) $\sin 30^\circ = \underline{\hspace{2cm}}$

b) $\cos 45^\circ = \underline{\hspace{2cm}}$

c) $60^\circ = \underline{\hspace{2cm}}$ radians

d) π radians = $\underline{\hspace{2cm}}$ $^\circ$

e) in quadrant IV, if $\cos x = 4/5$, then $\sin x = \underline{\hspace{2cm}}$

f) in quadrant III, if $\tan x = 3/4$, then $\cos x = \underline{\hspace{2cm}}$

g) $\sin(2\pi/3) = \underline{\hspace{2cm}}$

h) $\cos(\pi/2) = \underline{\hspace{2cm}}$

i) if $\tan x = 5/6$, then $\cot x = \underline{\hspace{2cm}}$

j) $\tan(\pi/3) = \underline{\hspace{2cm}}$

1 a) $\frac{1}{2}$

b) $\frac{\sqrt{2}}{2}$

c) $\frac{\pi}{6}$

d) 180°

e) $-\frac{3}{5}$

f) $-\frac{4}{5}$

g) $\frac{\sqrt{3}}{2}$

h) 0

i) $\frac{6}{5}$

j) $\sqrt{3}$

