Review Packet: Precalculus

Name ____________________

Period __________

Precalculus Semester 2 Mastery #1a)

1.) Given $w(q(x)) = f(x)$ find two possibilities for $w(x)$ and $q(x)$ where neither is equal to x.

$$f(x) = \sin^2(3x) - 7$$

$w(x) =$ ____________ $w(x) =$ ____________

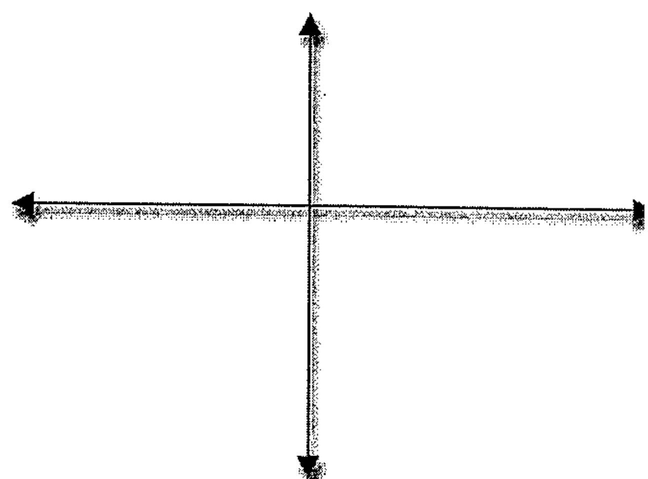
$q(x) =$ ____________ $q(x) =$ ____________

2.) Expand the following.

$$\log_4\left(\frac{xy^3}{16z}\right)$$

3.) Sketch a graph of the following. Find the intercepts and end behavior.

$$f(x) = -6x^3 - 17x^2 + 5x + 6$$

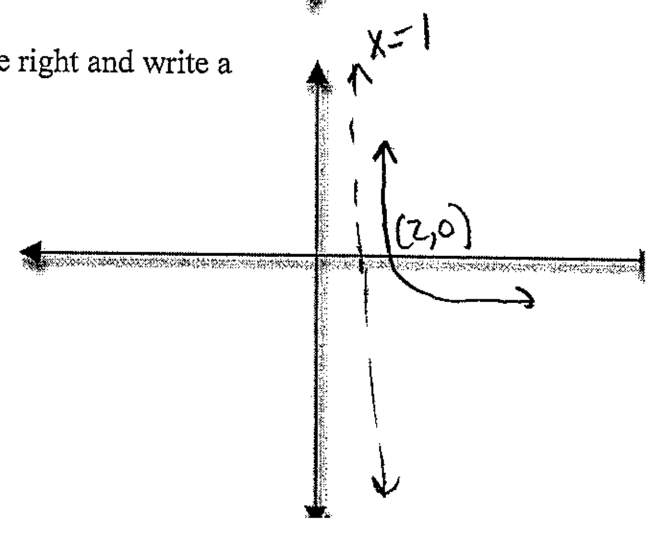


zeros:

y-intercept:

End behavior in limit notation

4.) Give the end behavior for the graph shown to the right and write a possible equation for the graph.



5.) Evaluate the value of each of the following. **Show the coordinate on the unit circle or draw the triangle.**

a.) $\cos\left(\frac{7\pi}{6}\right)$

b.) $\tan\left(\frac{2\pi}{3}\right)$

c.) $\csc\left(\frac{5\pi}{4}\right)$

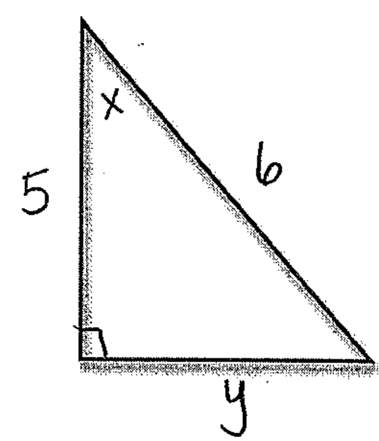
d.) $\cot\left(-\frac{7\pi}{4}\right)$

e.) $\sin\left(\frac{11\pi}{3}\right)$

f.) $\sec(\pi)$

6.) Find the $\tan\theta$ if $\sin\theta = \frac{2}{3}$ and $\tan\theta < 0$.

7.) Give the exact value for x and y shown in the diagram below.



8.) Find all value for $0 \le \theta \le 2\pi$ where $\sin\theta = \frac{\sqrt{3}}{2}$. Show how you found your solutions.

Name ____________________

Period _______

Mastery #2 a.) Precalculus

Solve the following equations. Give the exact solutions. Give logarithmic solutions in terms of *ln*.

1.) $7x^3 - 12x^2 = 4x$

2.) $4\left(\frac{1}{3}\right)^{2x-4} + 5 = 8$

3.) $\frac{1}{x} - \frac{5}{x+1} = 3$

4.) $(x-1)^{\frac{3}{2}} = -8$

5.) $6\sin(3x) - \sqrt{27} = 0$

6.) Find three different decompositions for the following where $q(k(x)) = h(x)$.

$$h(x) = \frac{5}{(4x+1)^2} - 3$$

7.) Evaluate the following. Give only the FUNCTION ANSWER!

a.) $\tan\left(-\frac{7\pi}{6}\right)$ b.) $\sin^{-1}(-1)$ c.) $\cos\left(\arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$

8.) Add the following- Find a common denominator.

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{x+h-x}$$

Name ______________________________

Period _____

Mastery #3a.)

1.) The volume of a box is given by the function $V(x) = x(10-2x)(14-2x)$, where x is the height of the box. For what heights will the box have a volume of 25 cubic units.

2.) Solve the equation. Find ALL solutions.

$$4\sin(2x) - \sqrt{8} = 0$$

3.) Sketch a graph of $f(x) = \sqrt{x}$. Find the average rate of change between x = 1 and x = 1 + h. Find the conjugate to estimate the tangent line slope at x = 1.

4.) Sketch a graph of $f(x) = x \sin x$ on the interval $[0, 2\pi]$. Find the following:

Increasing intervals

Decreasing intervals

Absolute mins

Relative mins

Absolute Maxs

Relative maxs

5.) Solve the equation: $\ln x - \ln(x-1) = 2$

Name ______________________________

Period _____

Mastery 4a.) Summative Assessment

1.) Condense the logarithm and then sketch a graph.

$f(x) = \ln(2x+2) - \ln 2$

x-intercept

y-intercept

Domain

Range

End Behavior in Limit Notation

2.) Sketch a graph of $f(x) = \dfrac{x^2}{x-1}$

End Behavior Asymptote

x-intercept

y-intercept

3.) The perimeter of rectangle is 100 feet. Set up a function that models the area of the rectangle. Sketch a graph of your model and find the maximum area possible.

4.) Sketch a graph of the trigonometric function.

$f(x) = 5\tan(2x) + 5$

Transformations

y-intercept

x-intercept

Domain

Range

5.) Verify the identity:

$$\frac{\sec x + 1}{\tan x} = \frac{\sin x}{1 - \cos x}$$

Name Key

Period ________

Precalculus Semester 2 Mastery #1a)

1.) Given $w(q(x)) = f(x)$ find two possibilities for $w(x)$ and $q(x)$ where neither is equal to x.

$$f(x) = \sin^2(3x) - 7$$

Answers vary

$w(x) =$ $\sin^2 x - 7$ $\quad$ $w(x) =$ $x^2 - 7$

$q(x) =$ $3x$ $\quad$ $q(x) =$ $\sin(3x)$

2.) Expand the following.

$$\log_4\left(\frac{xy^3}{16z}\right) = \log_4 x + 3\log_4 y - 2 - \log_4 z$$

3.) Sketch a graph of the following. Find the intercepts and end behavior.

$$f(x) = -6x^3 - 17x^2 + 5x + 6$$

$$\begin{array}{r|rrrr} 3 & -6 & -17 & 5 & 6 \\ & & 18 & -3 & -6 \\ \hline & -6 & 1 & 2 & 0 \end{array}$$

$-6x^2 + x + 2 = 0$

$(-3x+2)(2x+1) = 0$

$x = +\frac{2}{3}$ $\quad x = \frac{-1}{2}$

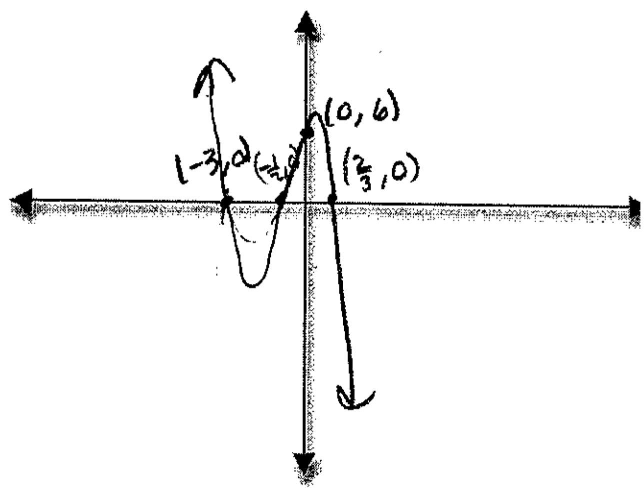
zeros: $(-3,0)\ (\frac{2}{3},0)\ (-\frac{1}{2},0)$

y-intercept: $(0,6)$

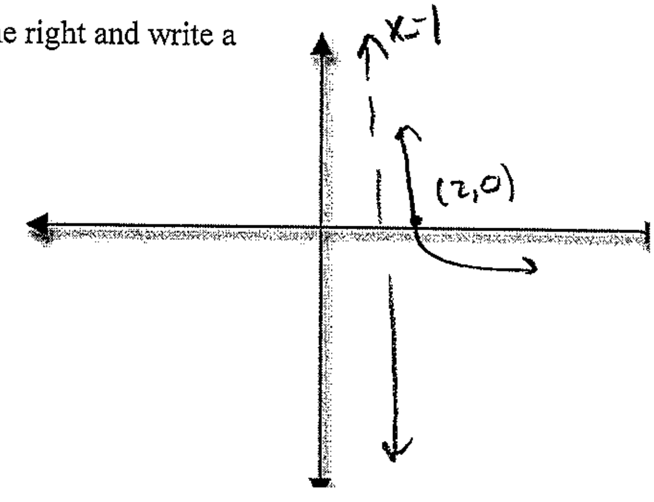
End behavior in limit notation

$\lim\limits_{x\to\infty} f(x) = -\infty$

$\lim\limits_{x\to-\infty} f(x) = \infty$



4.) Give the end behavior for the graph shown to the right and write a possible equation for the graph.



$$\boxed{y = \log_{\frac{1}{2}}(x-1)}$$

$\lim\limits_{x\to\infty} f(x) = -\infty$

$\lim\limits_{x\to1^+} f(x) = \infty$

5.) Evaluate the value of each of the following. **Show the coordinate on the unit circle or draw the triangle.**

a.) $\cos\left(\frac{7\pi}{6}\right) = \boxed{-\frac{\sqrt{3}}{2}}$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

b.) $\tan\left(\frac{2\pi}{3}\right)$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$= \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \boxed{-\sqrt{3}}$

c.) $\csc\left(\frac{5\pi}{4}\right) = \frac{-2\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{-2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

d.) $\cot\left(-\frac{7\pi}{4}\right) = \boxed{1}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

e.) $\sin\left(\frac{11\pi}{3}\right) - \frac{6\pi}{3} = \sin\left(\frac{5\pi}{3}\right)$

$\boxed{-\frac{\sqrt{3}}{2}}$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

f.) $\sec(\pi) = \frac{1}{-1} = \boxed{-1}$

$(-1, 0)$

6.) Find the $\tan\theta$ if $\sin\theta = \frac{2}{3}$ and $\tan\theta < 0$.

2, 3, $-\sqrt{5}$

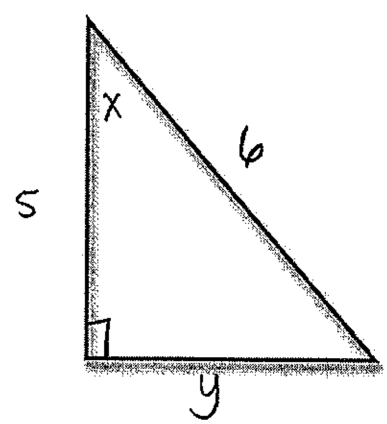
$a^2 + 4 = 9$

$a^2 = 5$

$a = \pm\sqrt{5}$

$\tan\theta = \frac{2\sqrt{5}}{-\sqrt{5}\sqrt{5}} = \boxed{\frac{-2\sqrt{5}}{5}}$

7.) Give the exact value for x and y shown in the diagram below.



$5^2 + y^2 = 6^2$

$25 + y^2 = 36$

$-25 \qquad -25$

$\sqrt{y^2} = \sqrt{11}$

$y = \pm\sqrt{11}$

$\boxed{y = \sqrt{11}}$

$\cos^{-1}\cos x = \cos^{-1}\frac{5}{6}$

$\boxed{x = \cos^{-1}\left(\frac{5}{6}\right)}$

8.) Find all value for $0 \le \theta \le 2\pi$ where $\sin\theta = \frac{\sqrt{3}}{2}$. Show how you found your solutions.

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

Name Key

Period ______

Mastery #2 a.) Precalculus

Solve the following equations. Give the exact solutions. Give logarithmic solutions in terms of *ln*.

1.) $7x^3 - 12x^2 = 4x$

$7x^3 - 12x^2 - 4x = 0$

$x(7x^2 - 12x - 4) = 0$

$x(7x+2)(x-2) = 0$

$\boxed{x = 0 \quad x = -\frac{2}{7} \quad x = 2}$

2.) $4\left(\frac{1}{3}\right)^{2x-4} + 5 = 8$

$\quad -5 \quad -5$

$\frac{4\left(\frac{1}{3}\right)^{2x-4}}{4} = \frac{3}{4}$

$\ln\left(\frac{1}{3}\right)^{2x-4} = \ln\frac{3}{4}$

$2x - 4 = \frac{\ln\frac{3}{4}}{\ln\frac{1}{3}} + 4$

$\quad +4$

$\boxed{x = \frac{\frac{\ln\frac{3}{4}}{\ln\frac{1}{3}} + 4}{2}}$

$\boxed{x = \frac{\ln\frac{3}{4}}{2\ln\frac{1}{3}} + 2}$

3.) $\frac{1}{x} - \frac{5}{x+1} = 3 \quad (x)(x+1)$

$x+1 - 5x = 3x^2 + 3x$

$-4x + 1 = 3x^2 + 3x$

$0 = 3x^2 + 7x - 1$

$x = \frac{-7 \pm \sqrt{49 - 4(3)(-1)}}{2(3)}$

$x = \frac{-7 \pm \sqrt{49 + 12}}{6}$

$\boxed{x = \frac{-7 \pm \sqrt{61}}{6}}$

4.) $(x-1)^{\frac{3}{2}} = -8$

$\sqrt[3]{\left(\sqrt{x-1}\right)^3} = \sqrt[3]{-8}$

$\sqrt{x-1} = -2$

Not possible

a square root

cannot = a negative #

$\boxed{\text{No Solution}}$

5.) $6\sin(3x) - \sqrt{27} = 0$

$\frac{6\sin 3x}{6} = \frac{\sqrt{27}}{6} = \frac{\sqrt{9}\sqrt{3}}{6}$

$\sin 3x = \frac{3\sqrt{3}}{6}$

$\sin 3x = \frac{\sqrt{3}}{2}$

$3x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\frac{3x}{3} = \frac{\pi}{3(3)} + \frac{2\pi n}{3}$

$\frac{3x}{3} = \frac{2\pi}{3(3)} + \frac{2\pi n}{3}$

$\boxed{x = \frac{\pi}{9} + \frac{2\pi n}{3}}$

$\boxed{x = \frac{2\pi}{9} + \frac{2\pi n}{3}}$

6.) Find three different decompositions for the following where $q(k(x)) = h(x)$.

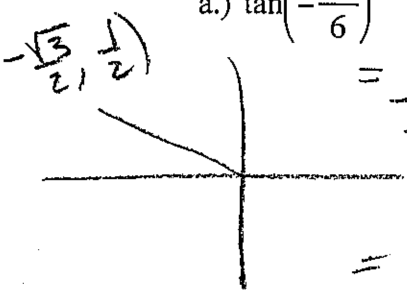
$$h(x) = \frac{5}{(4x+1)^2} - 3$$

$k(x) = 4x+1$ | $k(x) = (4x+1)^2$ | $k(x) = \frac{5}{(4x+1)^2}$

$g(x) = \frac{5}{x^2} - 3$ | $g(x) = \frac{5}{x} - 3$ | $g(x) = x - 3$

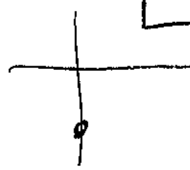
7.) Evaluate the following. Give only the FUNCTION ANSWER!

a.) $\tan\left(-\frac{7\pi}{6}\right)$



$= \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{-\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

b.) $\sin^{-1}(-1) = \boxed{-\frac{\pi}{2}}$



c.) $\cos\left(\arcsin\left(\frac{\sqrt{2}}{2}\right)\right) = \cos\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

8.) Add the following- Find a common denominator.

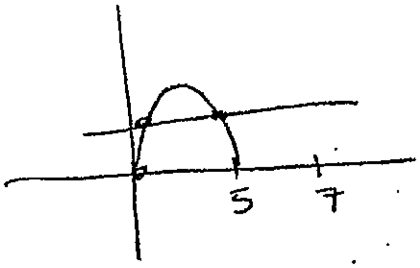
$$\frac{\frac{x}{x}\cdot\frac{1}{x+h} - \frac{1}{x}\cdot\frac{(x+h)}{(x+h)}}{x+h-x} = \frac{\frac{x-(x+h)}{x(x+h)}}{\cancel{x}+h-\cancel{x}} = \frac{\frac{\cancel{x}-\cancel{x}-h}{x(x+h)}}{\frac{h}{1}} = \frac{-\cancel{h}}{x(x+h)}\cdot\frac{1}{\cancel{h}}$$

$$= \boxed{\frac{-1}{x(x+h)}}$$

Name Key   Period ____

Mastery #3a.)

1.) The volume of a box is given by the function $V(x) = x(10-2x)(14-2x)$, where x is the height of the box. For what heights will the box have a volume of 25 cubic units?



$$\boxed{\begin{array}{l} x \approx 4.449 \text{ units} \\ x \approx 0.191 \text{ units} \end{array}}$$

2.) Solve the equation. Find ALL solutions.

$4\sin(2x) - \sqrt{8} = 0$

$\quad + \sqrt{8} \quad + \sqrt{8}$

$\dfrac{4\sin(2x)}{4} = \dfrac{2\sqrt{2}}{4}$

$\sin(2x) = \dfrac{\sqrt{2}}{2}$

$2x = \sin^{-1}\dfrac{\sqrt{2}}{2}$

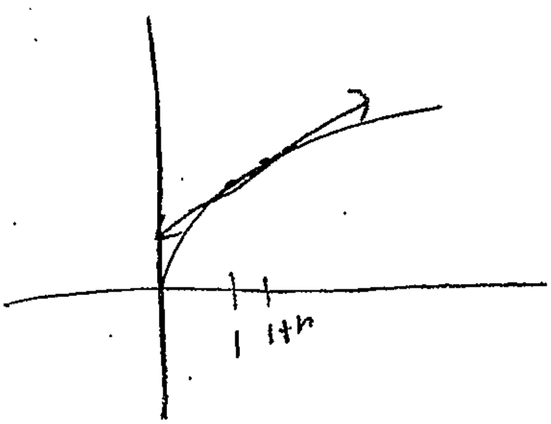
$\dfrac{2x}{2} = \dfrac{\pi}{2(4)} + \dfrac{2\pi n}{2}$

$\boxed{x = \dfrac{\pi}{8} + \pi n}$

$\dfrac{2x}{2} = \dfrac{3\pi}{2(4)} + \dfrac{2\pi n}{2}$

$\boxed{x = \dfrac{3\pi}{8} + \pi n}$

3.) Sketch a graph of $f(x) = \sqrt{x}$. Find the average rate of change between x = 1 and x = 1 + h. Find the conjugate to estimate the tangent line slope at x = 1.



$$\frac{\sqrt{1+h} - 1}{1+h-1} = \frac{(\sqrt{1+h} - 1)(\sqrt{1+h} + 1)}{h(\sqrt{1+h} + 1)}$$

secant line

$$= \frac{\cancel{1}+h \cancel{-1}}{h(\sqrt{h+1} + 1)} = \boxed{\frac{1}{\sqrt{h+1} + 1}}$$

$h \to 0$

so tangent line slope at $x = 1$

$$= \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

4.) Sketch a graph of $f(x) = x\sin x$ on the interval $[0, 2\pi]$. Find the following:

Increasing intervals $(0, 2.029)$ $(4.913, 2\pi)$

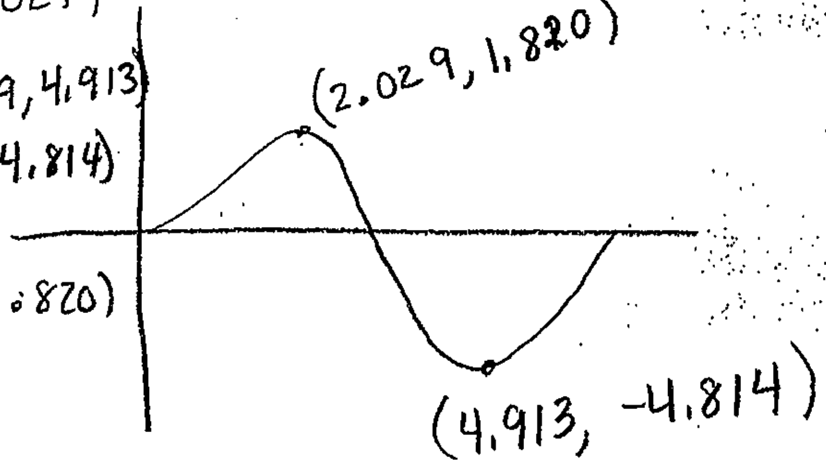
Decreasing intervals $(2.029, 4.913)$

Absolute mins $(4.913, -4.814)$

Relative mins none

Absolute Maxs $(2.029, 1.820)$

Relative maxs none



5.) Solve the equation: $\ln x - \ln(x-1) = 2$

$$e^{\ln\left(\frac{x}{x-1}\right)} = e^{2}$$

$$\frac{x}{\cancel{x-1}}\cancel{(x-1)} = e^2(x-1)$$

$$x = e^2x - e^2$$

$$x - e^2x = -e^2$$

$$\frac{x(1-e^2)}{1-e^2} = \boxed{\frac{-e^2}{1-e^2}} \approx \boxed{1.157}$$

Name Key

Period ____

Mastery 4a.) Formative Assessment

1.) Condense the logarithm and then sketch a graph.

$f(x) = \ln(2x+2) - \ln 2$ $\qquad f(x) = \ln\left(\frac{2x+2}{2}\right)$

$f(x) = \ln(x+1)$

x-intercept $(0,0)$

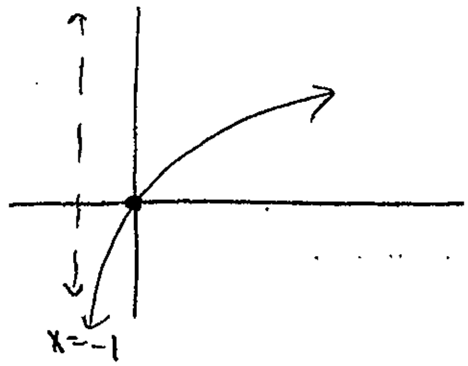
y-intercept $(0,0)$

Domain $\{x \mid x \in \mathbb{R}, x > -1\}$

Range $\{y \mid y \in \mathbb{R}\}$

End Behavior in Limit Notation

$\lim_{x\to\infty} f(x) = \infty \qquad \lim_{x\to -1^+} f(x) = -\infty$



$e^{0} = e^{\ln(x+1)}$

$1 = x+1$
$-1 \quad -1$

$0 = x \quad (0,0)$

$f(0) = \ln(1) = 0$
$(0,0)$

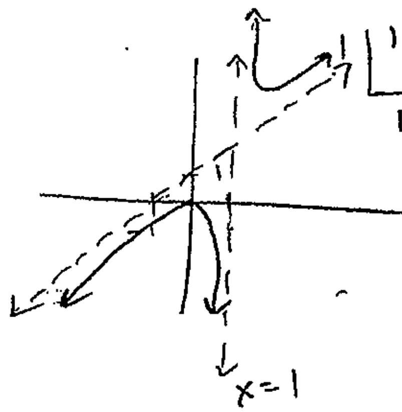
2.) Sketch a graph of $f(x) = \dfrac{x^2}{x-1}$

End Behavior Asymptote

$y = x+1$

x-intercept $(0,0)$

y-intercept $(0,0)$



$$\begin{array}{c|ccc} 1 & 1 & 0 & 0 \\ & & 1 & 1 \\ \hline & 1 & 1 & 1 \end{array}$$

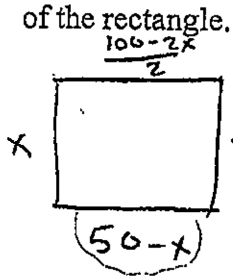
$y = x+1+\frac{1}{x-1}$

OR

$$\begin{array}{r} x+1+\frac{1}{x-1} \\ x-1 \overline{) \, x^2+0x+0} \\ -x^2+x \\ \hline x+0 \\ -x+1 \\ \hline 1 \end{array}$$

goes to 0 as $x \to \infty$

3.) The perimeter of rectangle is 100 [redacted] feet. Set up a function that models the area of the rectangle. Sketch a graph of your model and find the maximum area possible.



$A = xy$

$A = x(50-x)$

$0 = x(50-x)$

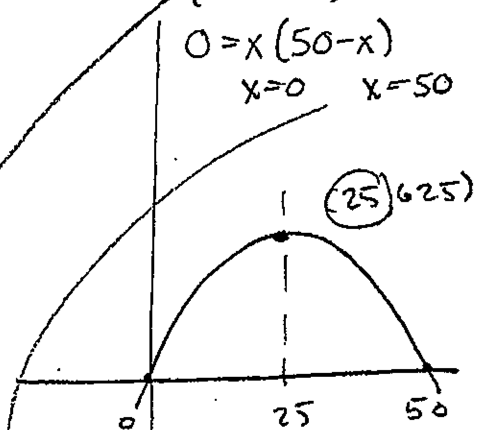
$x=0 \quad x=50$

$\boxed{\text{Max Area: } 625 \text{ ft}^2}$

$100 = 2x + 2y$
$-2x$

$\frac{100-2x}{2} = \frac{2y}{2}$

$50 - x = y$

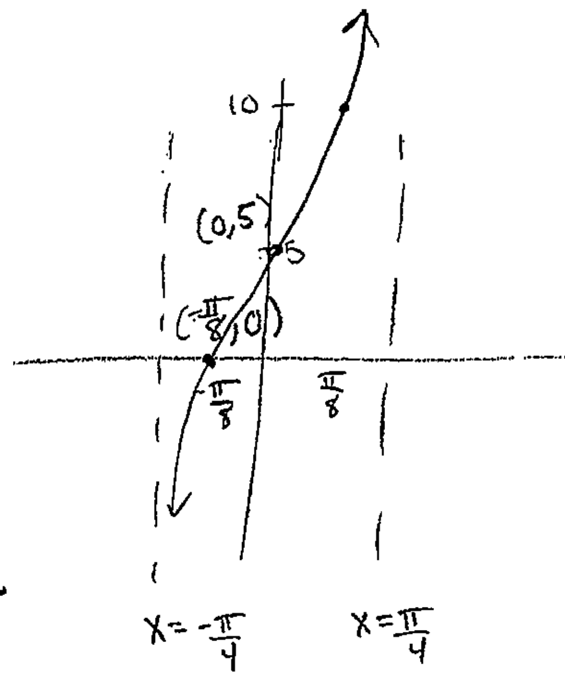


middle $x = 25$

$A = 25(25) = 625$

4.) Sketch a graph of the trigonometric function.

$f(x) = 5\tan(2x) + 5$



Transformations | v. stretch of 5 | h shrink of $\frac{1}{2}$ | v shift up 5

y-intercept

$f(0) = 5\tan(0) + 5 \quad f(0) = 5 \quad (0,5)$

x-intercept

$0 = 5\tan(2x) + 5 \quad \tan(2x) = -1$

$2x = -\frac{\pi}{4} + \pi n$

$\boxed{x = -\frac{\pi}{8} + \frac{\pi}{2} n}$

Domain

$\{x \mid x \in \mathbb{R},\ x \neq -\frac{\pi}{4} + \frac{\pi}{2} n\}$

Range

$\{y \mid y \in \mathbb{R}\}$

5.) Verify the identity:

$$\left(\frac{\sec x - 1}{\sec x - 1}\right) \frac{\sec x + 1}{\tan x} = \frac{\sin x}{1 - \cos x}$$

$$\frac{\sec^2 x - 1}{(\sec x - 1)\tan x}$$

$$\frac{\tan^2 x}{(\sec x - 1)(\tan x)}$$

$$\frac{\tan x}{\sec x - 1}$$

$$\tan x \left(\frac{1}{\sec x - 1}\right)$$

$$\frac{\sin x}{\cos x}\left(\frac{1}{\sec x - 1}\right)$$

$$\frac{\sin x}{1 - \cos x}$$

OR

$$\frac{\sin x(1 + \cos x)}{(1 - \cos x)(1 + \cos x)} = \frac{\sin x(1 + \cos x)}{1 - \cos^2 x}$$

$$= \frac{\sin x(1 + \cos x)}{\sin^2 x} = \frac{1 + \cos x}{\sin x}$$

$$= \frac{\frac{1}{\cos x} + \frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{\sec x + 1}{\tan x}$$

Name ____________________

Period __________

Average Rate of Change

Use the function $f(x) = x^2$ for the following.

1.) Sketch a graph of the function and find the slope of the secant line between $x = -2$ and $x = -1.9$. Sketch the secant line on your graph and write the equation of the secant line.

2.) Use $x = -2$ and $x = -2 + h$ to find the slope of the tangent line at $x = -2$. Draw the tangent line on your graph and write the equation of the tangent line.

3.) Sketch a graph of the function and find the slope of the secant line between $x = -1$ and $x = -.9$. Sketch the secant line on your graph and write the equation of the secant line.

4.) Use $x = -1$ and $x = -1 + h$ to find the slope of the tangent line at $x = -1$. Draw the tangent line on your graph and write the equation of the tangent line.

5.) Sketch a graph of the function and find the slope of the secant line between $x = 0$ and $x = 0.1$. Sketch the secant line on your graph and write the equation of the secant line.

6.) Use $x = 0$ and $x = 0 + h$ to find the slope of the tangent line at $x = 0$. Draw the tangent line on your graph and write the equation of the tangent line.

7.) Sketch a graph of the function and find the slope of the secant line between $x = 1$ and $x = 1.1$. Sketch the secant line on your graph and write the equation of the secant line.

8.) Use $x = 1$ and $x = 1 + h$ to find the slope of the tangent line at $x = 1$. Draw the tangent line on your graph and write the equation of the tangent line.

9.) What patterns do you notice? What would you guess is the slope of the tangent line at x = 2?

10.) Find $\dfrac{f(x+h) - f(x)}{(x+h) - x}$. What did this find?

Use the function $f(x) = 2x^2 - 8$ for the following.

11.) Sketch a graph of the function and find the slope of the secant line between $x = 5$ and $x = 5.01$. Sketch the secant line on your graph and write the equation of the secant line.

12.) Use $x = 5$ and $x = 5 + h$ to find the slope of the tangent line at $x = 5$. Draw the tangent line on your graph and write the equation of the tangent line.

Use the function $f(x) = -x^2 + 2x$ for the following.

13.) Sketch a graph of the function and find the slope of the secant line between $x = -3$ and $x = -3.001$. Sketch the secant line on your graph and write the equation of the secant line.

14.) Use $x = -3$ and $x = -3 + h$ to find the slope of the tangent line at $x = -3$. Draw the tangent line on your graph and write the equation of the tangent line.

7.) m = 2.1 y − 1 = 2.1(x − 1)

8.) y − 1 = 2(x − 1)

9.) The slope is double the x-value

10.) 2x

11.) m = 20.02 y − 42 = 20.02(x − 5)

12.) m = 20 y − 42 = 20(x − 5)

13.) m = 8.001 y + 15 = 8.001(x + 3)

Answers

1.) m = −3.9 y − 4 = −3.9(x − 2)

2.) m = −4 y − 4 = −4(x − 2)

3.) m = −1.9 y − 1 = −1.9(x − 1)

4.) m = −2 y − 1 = −2(x − 1)

5.) m = −.1 y − 0 = −.1(x…

6.) m = 0 y = 0x

Rational Functions Worksheet

Sketch a graph of each of the following, giving

- Non-removable discontinuities (vertical asymptotes)
- Removable discontinuities (holes)
- Intercepts
- End Behavior in limit notation/End Behavior asymptotes
- Intercepts
- Domain and Range

ANSWERS ARE ON THE BACK

1. $f(x)=\dfrac{x^2-1}{x+1}$
2. $f(x)=\dfrac{x^3+2x^2}{x}$
3. $f(x)=\dfrac{2x^2-7x-15}{x-5}$
4. $f(x)=\dfrac{x^3}{x^2-4}$
5. $f(x)=\dfrac{x+3}{x^2-9}$
6. $f(x)=\dfrac{x^2+4x+4}{x^3-4x}$
7. $f(x)=\dfrac{2x^3+2x^2-7x+3}{x-1}$
8. $f(x)=\dfrac{x+1}{x^2-4x}$
9. $f(x)=\dfrac{3x-1}{3x^2+20x-7}$
10. $f(x)=\dfrac{x^2-7x+12}{x-3}$
11. $f(x)=\dfrac{1}{x^2}-4$
12. $f(x)=\dfrac{3x^2+20x-7}{3x-1}$
13. $f(x)=2x+\dfrac{1}{x-1}$
14. $f(x)=\dfrac{x^2}{x+1}$
15. $f(x)=\dfrac{2x^2-8x}{x-3}$

16. Describe how to find vertical asymptotes. Describe how to find end behavior asymptotes.

17. What is the difference between a vertical asymptote and a horizontal asymptote?

18. What is the difference between a hole and a vertical asymptote?

19. Write AN equation of a rational function that has a removable discontinuity at x=-2, a x-intercept of (5,0), a non-removable discontinuity at x = 7, and a horizontal asymptote at y = 3.

20. Write AN equation of a rational function with a slant asymptote and a y intercept of (0,1).

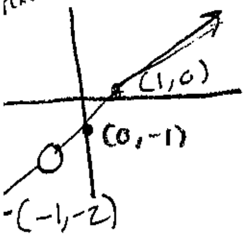
21. Write AN equation of a rational function that has a non-removable discontinuity of x=5 and x=-1/2, and has a removable discontinuity at x=6. What is the end behavior of your equation?

22. Write the equation of a rational function with a removable discontinuity at x = 1 and a non-removable discontinuity at x = -4. What is the end behavior asymptote of your function?

Answers 19 $f(x)=\dfrac{3(x-5)(x+2)}{(x-7)(x+2)}$ 20 $f(x)=\dfrac{x^2+1}{x+1}$ 21 $f(x)=\dfrac{(x-6)}{(x-5)(2x+1)(x-6)}$ $y=0$

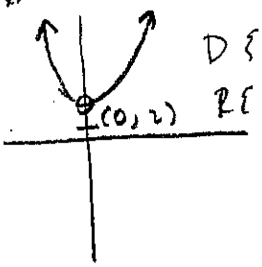
22 $f(x)=\dfrac{(x-1)}{(x-1)(x+4)}$ $y=0$

$f(x) = x - 1, \quad x \neq -1$



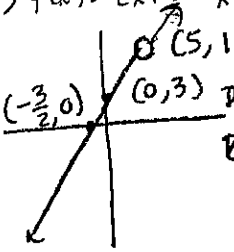
$D\{x \mid x \in \mathbb{R}, x \neq -1\}$
$R\{y \mid y \in \mathbb{R}, y \neq -2\}$

$f(x) = x^2 + 2, \quad x \neq 0$



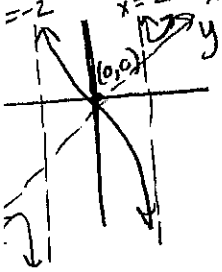
$D\{x \mid x \in \mathbb{R}, x \neq 0\}$
$R\{y \mid y \in \mathbb{R}, y > 2\}$

$f(x) = 2x + 3 \quad x \neq 5$



$D\{x \mid x \in \mathbb{R}, x \neq 5\}$
$R\{y \mid y \in \mathbb{R}, y \neq 13\}$

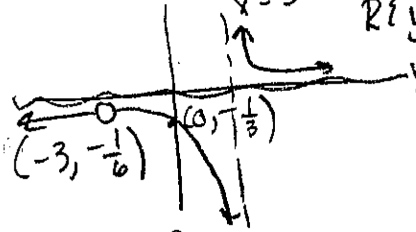
$f(x) = x + \frac{4x}{x^2-4}$



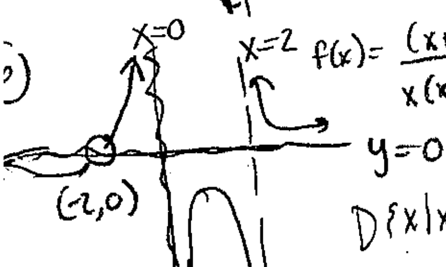
$D\{x \mid x \in \mathbb{R}, x \neq \pm 2\}$
$R\{y \mid y \in \mathbb{R}\}$

$f(x) = \frac{1}{x-3} \quad x \neq -3$

$D\{x \mid x \in \mathbb{R}, x \neq \pm 3\}$
$R\{y \mid y \in \mathbb{R}, y \neq -\frac{1}{6}, y \neq 0\}$

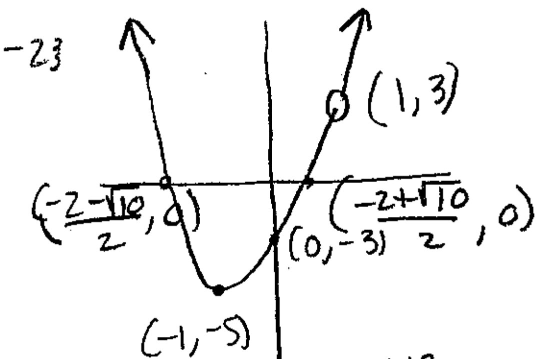


$f(x) = \frac{(x+2)}{x(x-2)} \quad x \neq -2$



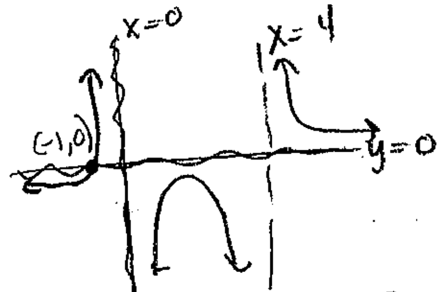
$D\{x \mid x \in \mathbb{R}, x \neq 0, \pm 2\}$

(7) $f(x) = 2x^2 + 4x - 3, \; x \neq 1$



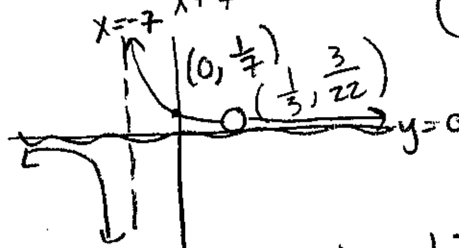
$D\{x \mid x \in \mathbb{R}, x \neq 1\}$
$R\{y \mid y \in \mathbb{R}, y \geq -5\}$

(8)



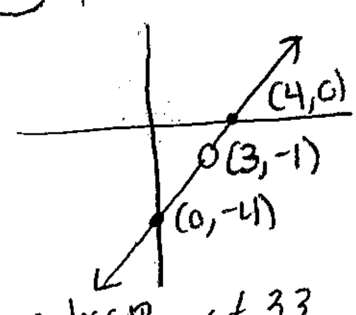
$D\{x \mid x \in \mathbb{R}, x \neq 0, 4\}$

(9) $f(x) = \frac{1}{x+7} \quad x \neq \frac{1}{3}$



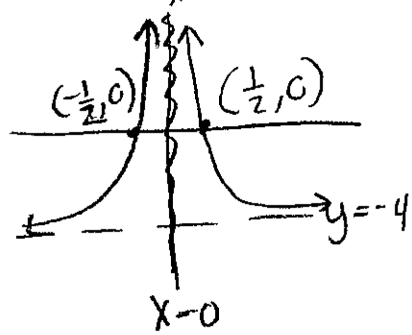
$D\{x \mid x \in \mathbb{R}, x \neq -7, \frac{1}{3}\}$
$R\{y \mid y \in \mathbb{R}, y \neq 0, \frac{3}{22}\}$

(10) $f(x) = x - 4 \quad x \neq 3$



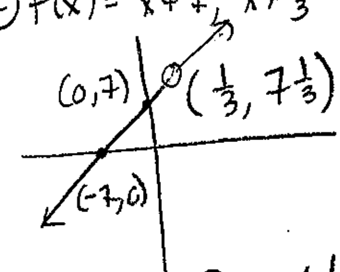
$D\{x \mid x \in \mathbb{R}, x \neq 3\}$
$R\{y \mid y \in \mathbb{R}, y \neq -1\}$

(11) $f(x) = \frac{1}{x^2} - 4$



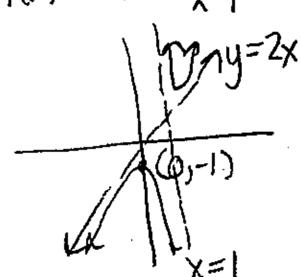
$D\{x \mid x \in \mathbb{R}, x \neq 0\}$
$R\{y \mid y \in \mathbb{R}, y > -4\}$

(12) $f(x) = x + 7, \; x \neq \frac{1}{3}$



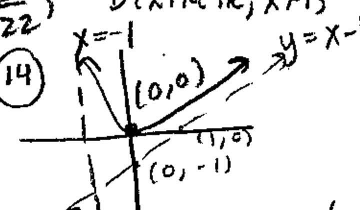
$D\{x \mid x \in \mathbb{R}, x \neq \frac{1}{3}\}$
$R\{y \mid y \in \mathbb{R}, y \neq 7\frac{1}{3}\}$

(13) $f(x) = 2x + \frac{1}{x-1}$



$D\{x \mid x \in \mathbb{R}, x \neq 1\}$

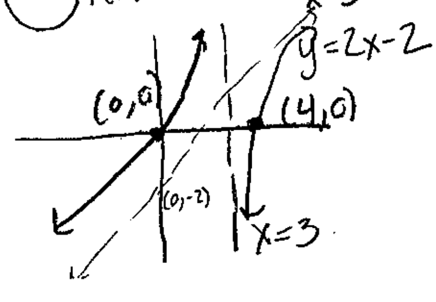
(14)



$D\{x \mid x \in \mathbb{R}, x \neq -1\}$

$f(x) = x - 1 + \frac{1}{x+1}$

(15) $f(x) = 2x - 2 - \frac{6}{x-3}$



Name ____________________

Period _____

Precalculus Review #1

1.) Find the average rate of change for $f(x)=\log_2 x$ between [4,16].

2.) Solve the following equations.

a.) $7x^3-12x^2-4x=0$

b.) $4\ln x-\ln 2=8$

c.) $\frac{1}{3}\cdot 2^{3x}-4=7$

d.) $\frac{1}{x+4}-2=8$

e.) $\cos 2x=1$

3.) Sketch a graph of each of the following. Give the end behavior in limit notation.

a.) $f(x)=\begin{cases} x^2-1 & x>0 \\ 3x & x\le 0 \end{cases}$

b.) $f(x)=\frac{2x+2}{x^2-1}$

c.) $f(x)=x^3+2x-3$

d.) $f(x)=-\ln x+1$

Precalculus Review #1

1.) $\dfrac{f(16)-f(4)}{16-4}$

$2^x=16 \quad 2^x=4$

$= \dfrac{\log_2 16 - \log_2 4}{12}$

$= \dfrac{4-2}{12}$

$= \dfrac{2}{12} = \boxed{\dfrac{1}{6}}$

2 a.) $7x^3-12x^2-4x=0$

$x(7x^2-12x-4)=0$

$(7x+2)(x-2)=0$

$\boxed{x=0 \quad x=\frac{-2}{7} \quad x=2}$

b.) $4\ln x - \ln 2 = 8$

$e^{\ln \frac{x^4}{2}} = e^8$

$(2)\dfrac{x^4}{2} = e^8(2)$

$\sqrt[4]{x^4} = \sqrt[4]{2e^8}$

$x = \sqrt[4]{2e^8} \approx \boxed{8.787}$

~~$x=-\sqrt[4]{2e^8}$~~ you cannot have a log of a negative #

c.) $\frac{1}{3}\cdot 2^{3x} - 4 = 7$

$\quad +4 \quad +4$

$\cancel{\frac{1}{3}}\cdot 2^{3x} = 11\cdot 3$

$\ln 2^{3x} = \ln 33$

$\dfrac{3x(\ln 2)}{3\ln 2} = \dfrac{\ln 33}{3\ln 2}$

$\boxed{x = \dfrac{\ln 33}{3\ln 2}} \approx \boxed{1.681}$

2 d.) $\dfrac{1}{x+4} - 2 = 8$

$\quad +2 \quad +2$

$\dfrac{1(x+4)}{x+4} = 10(x+4)$

$1 = 10x+40$

$-40 \quad -40$

$\dfrac{-39}{10} = \dfrac{10x}{10}$

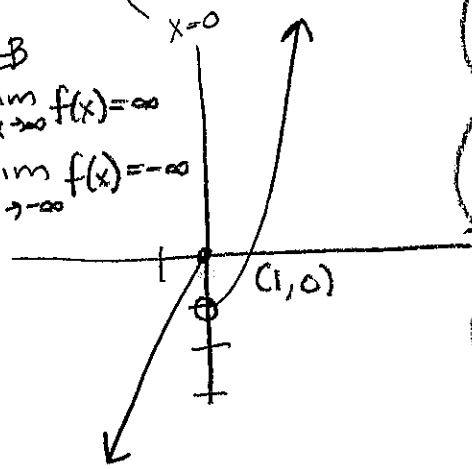
$\boxed{x=-3.9}$

3 a.) $f(x) = \begin{cases} x^2-1 & x>0 \\ 3x & x\le 0 \end{cases}$

EB

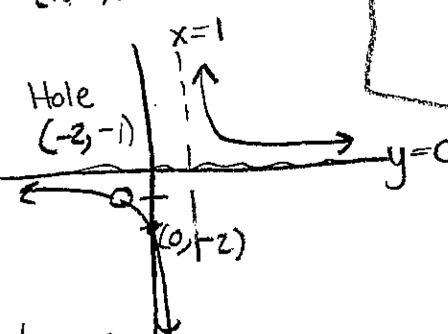
$\lim_{x\to\infty} f(x)=\infty$

$\lim_{x\to-\infty} f(x)=-\infty$



b) $f(x) = \dfrac{2x+2}{x^2-1}$

$f(x) = \dfrac{2\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} \quad x\ne -1$



$\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = 0$

3 c.) $f(x) = x^3+2x-3$

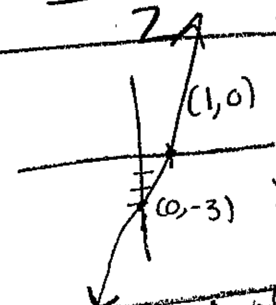
$$\begin{array}{r|rrrr} 1 & 1 & 0 & 2 & -3 \\ & & 1 & 1 & 3 \\ \hline & 1 & 1 & 3 & 0 \end{array} \quad (1,0)$$

$x^2+x+3=0$

$x = \dfrac{-1\pm\sqrt{1-4(1)(3)}}{2}$

$x = \dfrac{-1\pm\sqrt{1-12}}{2}$ Imaginary

$\boxed{x = \dfrac{-1\pm i\sqrt{11}}{2}}$



$\lim_{x\to\infty} f(x)=\infty$

$\lim_{x\to-\infty} f(x)=-\infty$

vertical reflection, up 1

3 d.) $f(x) = -\ln x + 1$

x int $\quad 0 = -\ln x + 1$

$\quad -1 \quad -1$

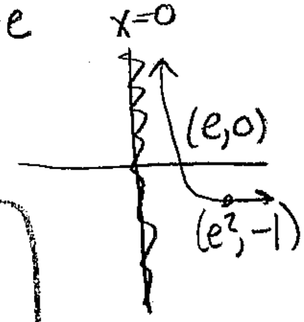
$\dfrac{-1}{-1} = \dfrac{-\ln x}{-1}$

$e^1 = e^{\ln x}$

$x = e$

$\lim_{x\to 0^+} f(x)=\infty$

$\lim_{x\to\infty} f(x)=-\infty$



$x=e^2$

$f(e^2) = -\ln e^2 + 1$

$= -2+1$

$= -1$

Precalculus Review #2

1.) Sketch a graph of $f(x) = \begin{cases} 2x+4 & x \geq 0 \\ 5 & x < 0 \end{cases}$. Label all intercepts, the domain and the range.

2.) Find the end behavior in limit notation of $f(x) = -5\left(\frac{1}{2}\right)^x - 4$.

3.) Solve the equation: $2x^4 - x^3 + x^2 - x - 1 = 0$

4.) Rewrite the following as a natural logarithm. $\log_2 7$

5.) Evaluate each of the following if possible:

a.) $\ln e^2$ b.) $\ln 1$ c.) $\ln 0$

Name ______________________

Precalculus Review #3

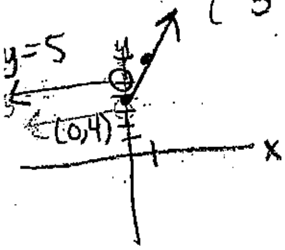
Solve each equation. Give **all** solutions and simplify your solutions.

1. $2x^3 + 3x^2 - 8x = -3$
2. $3^{x-1} + 4 = 8$
3. $\frac{1}{x+1} - \frac{2}{x-4} = \frac{5}{x^2 - 3x - 4}$
4. $\log_2 \frac{1}{2} + \log_2(x-1) = 2$
5. $-4\cot\left(\frac{x}{3}\right) + 4\sqrt{3} = 0$
6. Long divide the following:

$$\frac{x^3 - 5x^2 + 8x - 3}{x^2 + 1}$$

Key

1.) $f(x)=\begin{cases}2x+4 & x\ge 0\\ 5 & x<0\end{cases}$



yint (0,4)

xint none

D $\{x \mid x\in\mathbb{R}\}$

R $\{y \mid y\in\mathbb{R},\ y\ge 4\}$

3.) $2x^4-x^3+x^2-x-1=0$

$\boxed{x=1}$ | 2 -1 1 -1 -1

2 1 2 1

2 1 2 1 0

$2x^3+x^2+2x+1=0$

$x^2(2x+1)+1(2x+1)=0$

$(x^2+1)(2x+1)=0$

$x^2+1=0 \qquad 2x+1=0$

$-1 \quad -1 \qquad -1 \quad -1$

$x^2=-1 \qquad \frac{2x}{2}=\frac{-1}{2}$

$\boxed{x=\pm i} \qquad \boxed{x=-\frac{1}{2}}$

5a) $\ln e^2 = 2\ln e = 2 = \boxed{2}$

b) $\ln 1 = \boxed{0}$

$e^? = 1$

c) $\ln 0$

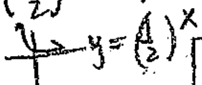
$e^? = 0$

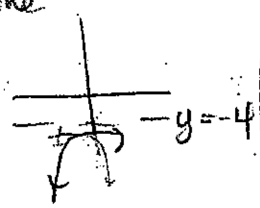
Not possible

2.) $f(x)=-5\left(\frac{1}{2}\right)^x-4$

V: flip, stretch of 5, down 4

H: none





$\lim_{x\to\infty} f(x) = -4$

$\lim_{x\to-\infty} f(x) = -\infty$

4) $\log_2 7 = \boxed{\frac{\ln 7}{\ln 2}}$

Key

1.) $2x^3+3x^2-8x+3=0$

1 | 2 3 -8 3

2 5 -3

2 5 -3 0

$2x^2+5x-3$

$(2x-1)(x+3)=0$

$\boxed{x=\frac{1}{2},\ x=-3,\ x=1}$

2) $3^{x-1}+4=8$

$-4 \quad -4$

$\ln 3^{x-1}=\ln 4$

$x-1(\ln 3)=\ln 4$

$x-1=\frac{\ln 4}{\ln 3}+1$

$+1$

$\boxed{x=\frac{\ln 4}{\ln 3}+1}$

3) $\left(\frac{1}{x+1}-\frac{2}{x-4}=\frac{5}{(x-4)(x+1)}\right)(x+1)(x-4)$

$\boxed{x\ne -1,\ x\ne 4}$

$1(x-4)-2(x+1)=5$

$x-4-2x-2=5$

$-x-6=5$

$+6 \quad +6$

$\frac{-1x}{-1}=\frac{11}{-1}$

$\boxed{x=-11}$

4.) $\log_2\frac{1}{2}+\log_2(x-1)=2$

$-1+\log_2(x-1)=2$

$+1 \qquad +1$

$2^{\log_2(x-1)}=2^3$

$x-1=8$

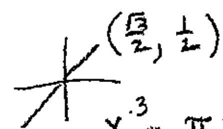
$+1 \quad +1$

$\boxed{x=9}$

5) $-4\cot\left(\frac{x}{3}\right)+4\sqrt{3}=0$

$\frac{-4\cot\left(\frac{x}{3}\right)}{-4}=\frac{-4\sqrt{3}}{-4}$

$\cot\left(\frac{x}{3}\right)=\sqrt{3}$



$\frac{x}{3}\cdot 3=\frac{\pi}{6}\cdot 3+\pi n\cdot 3$

$\boxed{x=\frac{\pi}{2}+3\pi n}$

6) $\boxed{x-5+\frac{7x+2}{x^2+1}}$

$x^2+1 \overline{\big)\, x^3-5x^2+8x-3}$

$-x^3 \qquad +x$

$-5x^2+7x-3$

$+5x^2 \qquad +5$

$7x+2$

Precalculus Review #4

Simplify the following expressions so that you can graph them. Then graph the expressions.

1.) $\dfrac{x^{\frac{1}{2}}+x}{\sqrt{x}}$

2.) $\dfrac{x}{x-1}-\dfrac{1}{x}$

3.) $\dfrac{\sin^2 x}{1-\cos x}$

4.) $\dfrac{-\sqrt{20}(x-1)^{\frac{1}{3}}}{\sqrt{45}}$

5.) $\ln x - \ln e^4$

6.) $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}$

7.) $\sqrt{\dfrac{1}{9}-\dfrac{x}{9}}$

Precalculus Review #4

Simplify the following expressions so that you can graph them. Then graph the expressions.

1.) $\dfrac{x^{\frac{1}{2}}+x}{\sqrt{x}}$

2.) $\dfrac{x}{x-1}-\dfrac{1}{x}$

3.) $\dfrac{\sin^2 x}{1-\cos x}$

4.) $\dfrac{-\sqrt{20}(x-1)^{\frac{1}{3}}}{\sqrt{45}}$

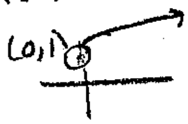
5.) $\ln x - \ln e^4$

6.) $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}$

7.) $\sqrt{\dfrac{1}{9}-\dfrac{x}{9}}$

1.) $\frac{\cancel{\sqrt{x}}(1+\sqrt{x})}{\cancel{\sqrt{x}}}$

$f(x)=1+\sqrt{x}$



emovable discontinuity (0,1)

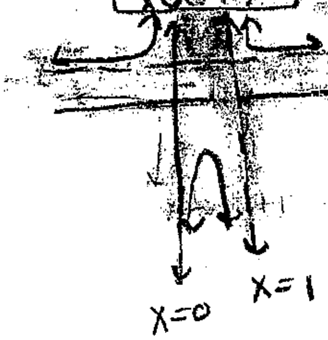
2.) $\frac{x(x)}{(x-1)(x)} - \frac{1(x-1)}{x(x-1)}$

$\boxed{\frac{x^2-x+1}{x(x-1)}}$

$x=\frac{1\pm\sqrt{1-4(1)(1)}}{2}$

$x=\frac{1\pm\sqrt{-3}}{2}$

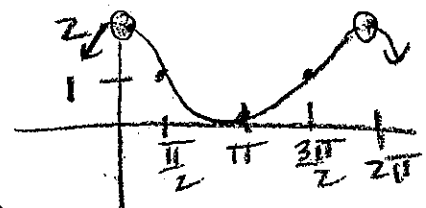
imaginary



3.) $\frac{\sin^2 x}{1-\cos x} = \frac{1-\cos^2 x}{1-\cos x}$

$=\frac{(\cancel{1-\cos x})(1+\cos x)}{\cancel{1-\cos x}}$ $\quad 1-\cos x$ $\quad x\neq 2\pi n$

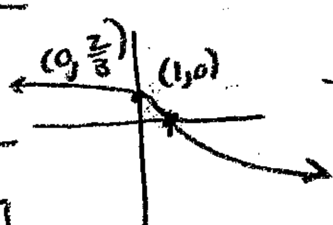
$=1+\cos x$



4.) $\frac{-\sqrt{20}\sqrt[3]{x-1}}{\sqrt{45}}$

$\frac{-2\sqrt{5}\sqrt[3]{x-1}}{3\sqrt{5}}$

$=\boxed{-\frac{2}{3}\sqrt[3]{x-1}}$



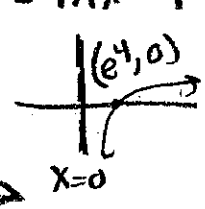
5.) $\ln x - \ln e^4$

$=\ln x - 4$

$0=\ln x - 4$

$4=\ln x$

$x=e^4$



6.) $\frac{(\cos x)\cos x}{(\cos x)(1+\sin x)} + \frac{(1+\sin x)(1+\sin x)}{\cos x(1+\sin x)}$

$\frac{\cos^2 x+1+2\sin x+\sin^2 x}{\cos x(1+\sin x)}$

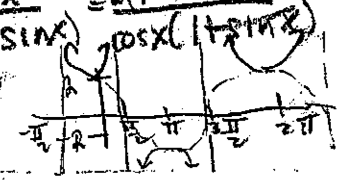
$1+\sin x=0$

$\sin x=-1$

$x\neq\frac{3\pi}{2}+2$

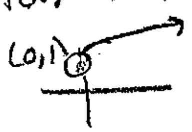
$=\frac{2+2\sin x}{\cos x(1+\sin x)} = \frac{2\cancel{(1+\sin x)}}{\cos x\cancel{(1+\sin x)}}$

$=2\sec x$



.) $\frac{\cancel{\sqrt{x}}(1+\sqrt{x})}{\cancel{\sqrt{x}}}$

$f(x)=1+\sqrt{x}$



movable discontinuity (0,1)

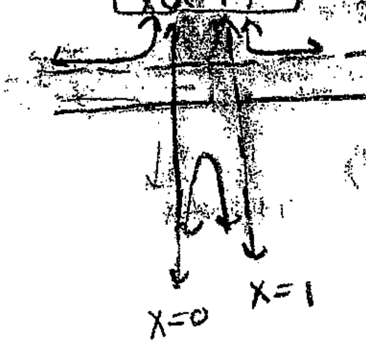
2.) $\frac{x(x)}{(x-1)(x)} - \frac{1(x-1)}{x(x-1)}$

$\boxed{\frac{x^2-x+1}{x(x-1)}}$

$x=\frac{1\pm\sqrt{1-4(1)(1)}}{2}$

$x=\frac{1\pm\sqrt{-3}}{2}$

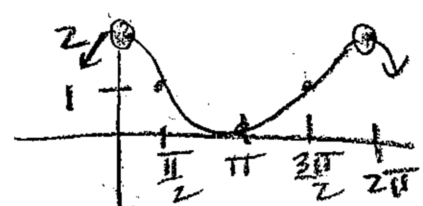
imaginary



3.) $\frac{\sin^2 x}{1-\cos x} = \frac{1-\cos^2 x}{1-\cos x}$

$=\frac{(\cancel{1-\cos x})(1+\cos x)}{\cancel{1-\cos x}}$ $\quad 1-\cos x$ $\quad x\neq 2\pi n$

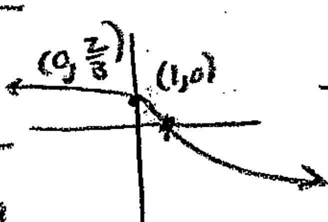
$=1+\cos x$



.) $\frac{-\sqrt{20}\sqrt[3]{x-1}}{\sqrt{45}}$

$\frac{-2\sqrt{5}\sqrt[3]{x-1}}{3\sqrt{5}}$

$\boxed{-\frac{2}{3}\sqrt[3]{x-1}}$



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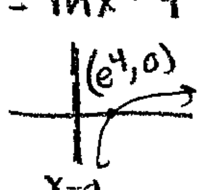

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$\frac{\cos^2 x+1+2\sin x+\sin^2 x}{\cos x(1+\sin x)}$

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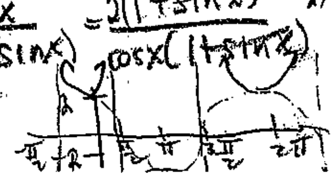


Key

Review Guide for College Algebra & Trigonometry

I) **Functional notation.** Given that $f(x) = 3x^2 - 2x + 5$ and $g(x) = 3x - 1$, find each of the following:

1) $g(4) - f(-2) = (3(4) - 1) - (3(-2)^2 - 2(-2) + 5)$

$= (12 - 1) - (3(4) + 4 + 5) = 11 - (12 + 9) = 11 - 21 = \boxed{-10}$

2) $g(x + h) - g(x) = (3(x+h) - 1) - (3x - 1) = 3x + 3h - 1 - 3x + 1 = \boxed{3h}$

3) $(g \circ f)(x) = g(f(x)) = 3(3x^2 - 2x + 5) - 1 = 9x^2 - 6x + 15 - 1 = \boxed{9x^2 - 6x + 14}$

4) $g^{-1}(x)$ $\quad x = 3y - 1$; $+1 \quad +1$; $\frac{x+1}{3} = \frac{3y}{3}$ $\quad \boxed{g^{-1}(x) = \frac{x+1}{3}}$

II) **Linear inequalities in one variable.** Solve. Graph on a number line and write in interval notation.

1) $3(x - 4) - (x + 1) \le x - 12$

$3x - 12 - x - 1 \le x - 12$

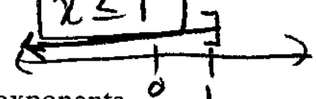
$2x - 13 \le x - 12$

$-x \quad\quad -x$

$x - 13 \le -12$

$+13 \quad +13$

$\boxed{x \le 1}$ $\quad (-\infty, 1]$



III) **Exponents and polynomials.** Simplify. Write all answers with positive exponents.

1) $(3x - 5)^4 = (3x-5)(3x-5)(3x-5)(3x-5) = (9x^2 - 30x + 25)(9x^2 - 30x + 25)$

$9x^2 - 15x - 15x + 25$

$81x^4 - 270x^3 + 225x^2 - 270x^3 + 900x^2 - 750x + 225x^2 - 750x + 625$

$\boxed{81x^4 - 540x^3 + 1350x^2 - 1500x + 625}$

2) $(4x^2y^6z)^2(-x^5y^7z^8)^6 = 16x^4y^{12}z^2 \cdot x^{30}y^{42}z^{48}$

$\boxed{16x^{34}y^{54}z^{50}}$

3) $$\frac{4x^4 - 4x^3 + 3x^2 - 5x + 2}{2x - 3}$$

$\boxed{2x^3 + x^2 + 3x + 2 + \frac{8}{2x-3}}$

$2x - 3 \,\big)\, 4x^4 - 4x^3 + 3x^2 - 5x + 2$

$-4x^4 \oplus 6x^3$

$2x^3 + 3x^2 - 5x + 2$

$\ominus 2x^3 \oplus 3x^2$

$6x^2 - 5x + 2$

$\ominus 6x^2 + 9x$

$4x + 2$

$-4x + 6$

8

IV) **Complex Numbers.** Simplify.

1) $\sqrt{-25}\sqrt{-81}$

$5i \cdot 9i = 45i^2 = 45(\sqrt{-1})^2 = 45(-1) = \boxed{-45}$

2) $\dfrac{(3 - 4i)(6 - 2i)}{(6 + 2i)(6 - 2i)} = \dfrac{18 - 6i - 24i + 8i^2}{36 + 12i - 12i - 4i^2}$

$+4$

$= \dfrac{18 - 30i - 8}{40}$

$= \dfrac{10}{40} - \dfrac{30i}{40} = \boxed{\dfrac{1}{4} - \dfrac{3}{4}i}$

3) $(5 + 2i)(-3 - 5i)$

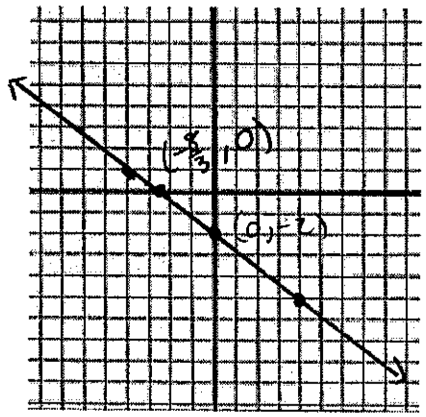
$-15 - 25i - 6i - 10i^2$

$= -15 - 31i + 10$

$= \boxed{-5 - 31i}$

V) Equations and inequalities in two variables. Graph on a coordinate system. Identify the intercepts and the slope of the linear equation.

1) $y = (-3/4)x - 2$



2) Solve the system:

A. $2x - 2y - z = -1$
B. $x + 2y + 2z = 9$
C. $x + y - z = 8$

(-C+B)
(-2C+A)

$2x - 2y - z = -1$
$x + 2y + 2z = 9$
$-x - y + z = -8$
$-4y + z = -17$
$y + 3z = 1$

$\longrightarrow$

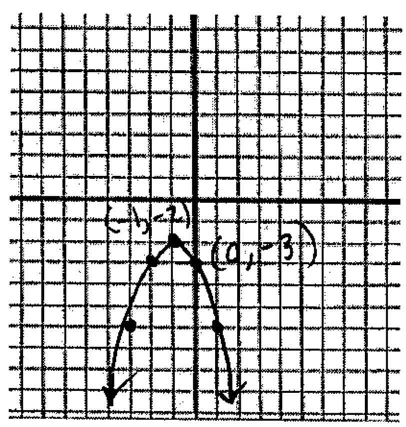
$-4y + z = -17$
$4y + 12z = 4$
$13z = -13$

$z = -1$
$x + 2y - z = 9$
$x + y + 1 = 8$
$\downarrow$
$x + 2y = 11$
$x + y = 7$
$\downarrow$
$x + 2y = 11$
$-x - y = -7$
$y = 4$
$x = 3$

$\boxed{x = 3 \quad y = 4 \quad z = -1}$

VI) Quadratic equations and functions. Graph on a coordinate system. Identify the coordinates of the vertex and the intercepts.

1) $g(x) = -(x + 1)^2 - 2$



VII) Rational functions and expressions.

1) Find the domain of

$$f(a) = \frac{a^2 + 2a - 3}{3a^2 + 11a + 6}$$

$\{x \mid x \in \mathbb{R}, x \neq -\frac{2}{3}, x \neq -3\}$

$$f(a) = \frac{(a+3)(a-1)}{(3a+2)(a+3)}$$

$$\frac{3p}{(p+2)(p+3)} = \frac{5p}{(p+3)(p-1)} - \frac{2}{(p+2)(p-1)}$$

LCD: $(p-1)(p+2)(p+3)$

$3p(p-1) = 5p(p+2) - 2(p+3)$

2) Solve: $3p^2-3p = 5p^2+10p-2p-6$

$$\frac{3p}{p^2+5p+6} = \frac{5p}{p^2+2p-3} - \frac{2}{p^2+p-2}$$

$3p^2-3p = 5p^2+8p-6$

$0 = 2p^2+11p-6$

$0 = (2p-1)(p+6)$

$0 = 2p-1 \quad p+6=0$

$\boxed{p=\frac{1}{2} \quad p=-6}$

VIII) Logarithms and exponentials.

1) Condense into a single log and simplify: log(10) - log(5) $\log\left(\frac{10}{5}\right)$

$\boxed{\log 2}$

2) Expand into sums, differences, and products.

$$\log\left(\frac{abc^3}{d^2}\right)$$

$\log(abc^3) - \log(d^2)$

$\log a + \log b + \log c^3 - \log d^2$

$\boxed{\log a + \log b + 3\log c - 2\log d}$

3) Solve:

a) $\ln x + \ln(2x+1) = 0$

$\ln(2x^2+x) = 0$

$2x^2+x=1$

$2x^2+x-1=0$

$(2x-1)(x+1)=0$

$x=\frac{1}{2} \quad \cancel{x=-1}$

b) $9^x = 27$ $\quad (3^2)^x = 3^3$

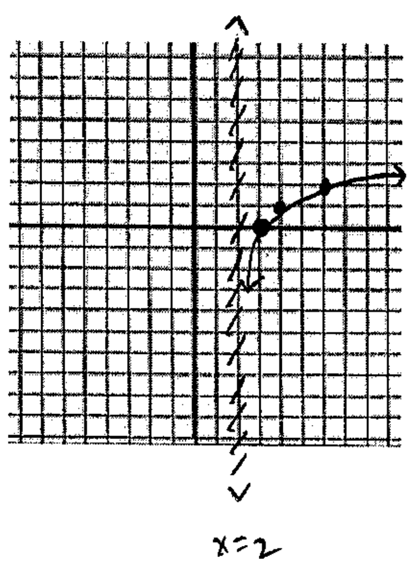
$3^{2x} = 3^3$

$2x=3$

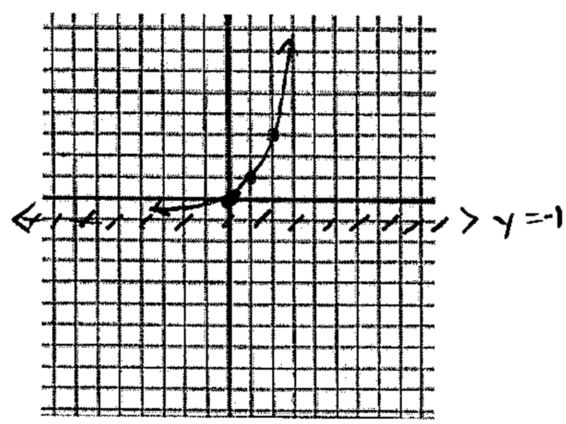
$\boxed{x=\frac{3}{2}}$

4) Graph:

a) $g(x) = \log_2(x-2)$

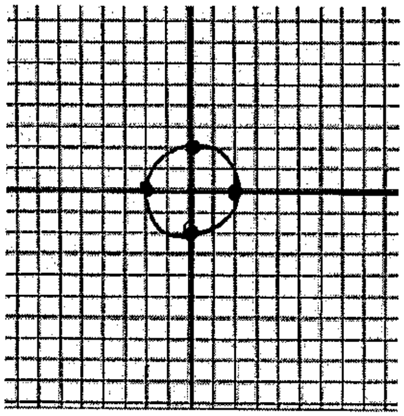


b) $p(x) = 2^x - 1$



IX) **Miscellaneous graphing.** Graph on a coordinate system.

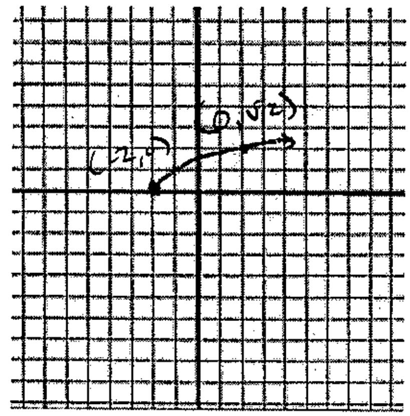
1) $x^2 + y^2 = 4$



center
(0,0)
x ints:
(-2,0)
(2,0)
y ints:
(0,2)
(0,-2)

2) $f(x) = \sqrt{x+2}$ *for* $x \geq -2$

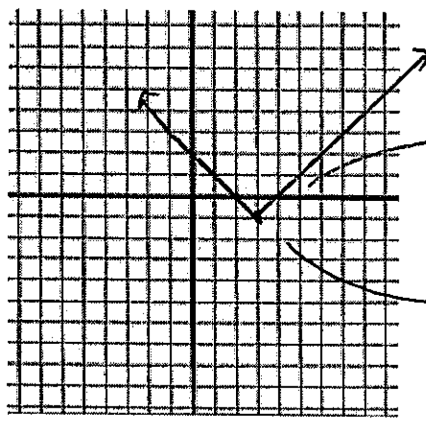
x int:
(-2,0)



y int:
(0, √2)

3) $f(x) = |x - 3| - 1$

y int:
(0,2)



x-ints: (2,0) (4,0)

(3,-1)

X) **Polynomial functions.**

1) Find all zeros of the function: $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

$$\begin{array}{r|rrrrr} 1 & -2 & 13 & -21 & 2 & 8 \\ & & -2 & 11 & -10 & -8 \\ \hline 2 & -2 & 11 & -10 & -8 & 0 \\ & & -4 & 14 & 8 & \\ \hline & -2 & 7 & 4 & 0 & \end{array}$$

$f(x) = (x-1)(x-2)(-2x^2+7x+4)$
$= (x-1)(x-2)(2x^2-7x-4)$
$= -(x-1)(x-2)(2x+1)(x-4)$

$\boxed{x=1 \quad x=2 \quad x=-\frac{1}{2} \quad x=4}$

XI) **Matrices and determinants.**

1) Evaluate the determinant of

$$\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}$$

$(3)(5) - (2)(4)$
$15 - 8$
$\boxed{7}$

XII) Sequences and series.

1) Write the first five terms of the arithmetic sequence

$A_1 = 6, \quad A_{k+1} = A_k - 5$

$6, 1, -4, -9, -14$

2) Find the n^{th} partial sum of the arithmetic sequence

$\overset{+12}{8}, \overset{+12}{20}, 32, 44, \ldots \qquad n = 10$

$a_n = 12n - 4$

$$\sum_{n=1}^{10}(12n-4) = \frac{(8 + 120-4)10}{2} = \frac{1240}{2} = 620$$

3) Write the first five terms of the geometric sequence $a_1 = 1, \; r = 1/3$

$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}$

3) Find the given sum. Round your answer to the nearest tenth.

$$\sum_{k=1}^{10} 8\left(\frac{-1}{4}\right)^{k-1} = \frac{8\left(1-\left(-\frac{1}{4}\right)^{10}\right)}{1+\frac{1}{4}} \approx 6.4$$

XIII) Trigonometry.

1) Find the indicated values.

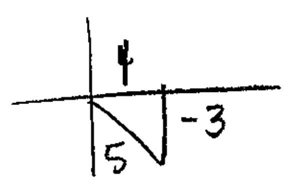
a) sin 30° = $\sin\frac{\pi}{10}$ = $\boxed{\frac{1}{2}}$

b) cos 45° = $\boxed{\frac{\sqrt{2}}{2}}$

c) 60° = $\boxed{\frac{\pi}{3}}$ radians

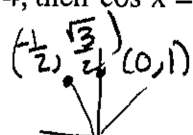
d) π radians = $\boxed{180°}$

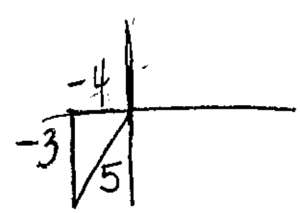
e) in quadrant IV, if cos x = 4/5, then sin x = $\boxed{-\frac{3}{5}}$



f) in quadrant III, if tan x = 3/4, then cos x = $\boxed{-\frac{4}{5}}$

g) sin(2π/3) = $\boxed{\frac{\sqrt{3}}{2}}$

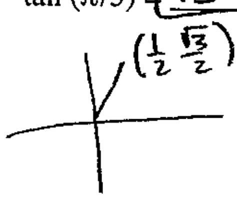




h) cos(π/2) = $\boxed{0}$

i) if tan x = 5/6, then cot x = $\boxed{\frac{6}{5}}$

j) tan (π/3) = $\boxed{\sqrt{3}}$



$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

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2) $g(x + h) - g(x)$

3) $(g \circ f)(x)$

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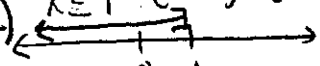
1)
$$\sqrt{-25}\,\sqrt{-81}$$

2)
$$\frac{3-4i}{6+2i}$$

3)
$$(5+2i)(-3-5i)$$

Key I) 1) -10 2) $3h$ 3) $9x^2 - 6x + 14$ 4) $\frac{x+1}{3}$ II) $x \leq 1$ $(-\infty, 1]$



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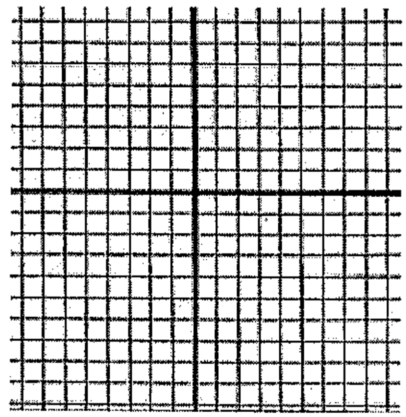

2) Solve the system:

$$\begin{aligned} 2x - 2y - z &= -1 \\ x + 2y + 2z &= 9 \\ x + y - z &= 8 \end{aligned}$$

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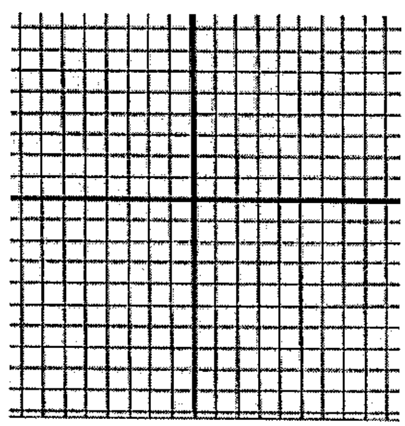

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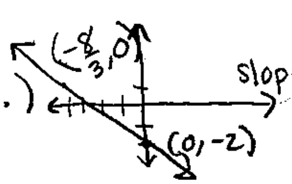
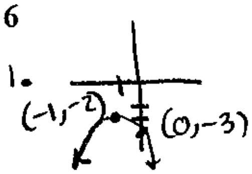
V) 1.)


slope $= -\frac{3}{4}$ 2) $(3,4,-1)$ VI) 1.


VII) $\{a \mid a \in \mathbb{R}, a \neq -\frac{2}{3}, a \neq -3\}$

2) Solve:

$$\frac{3p}{p^2+5p+6}=\frac{5p}{p^2+2p-3}-\frac{2}{p^2+p-2}$$

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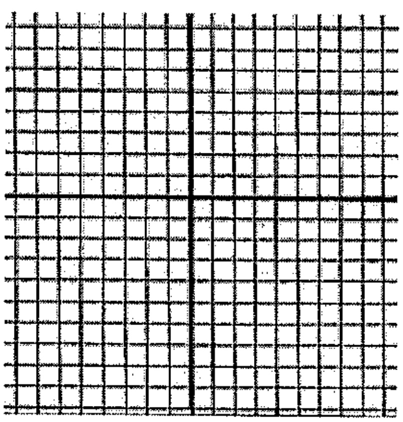
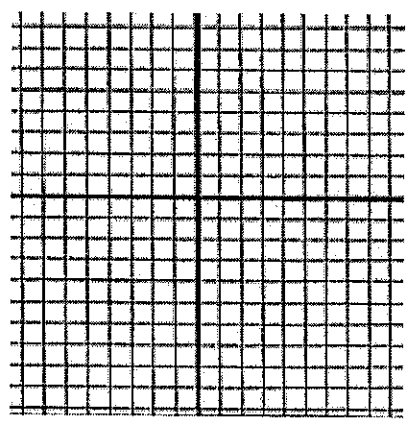
4) Graph:

a) $g(x) = \log_2 (x - 2)$

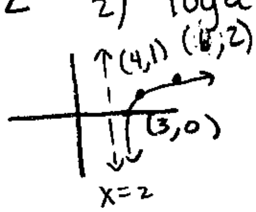


b) $p(x) = 2^x - 1$

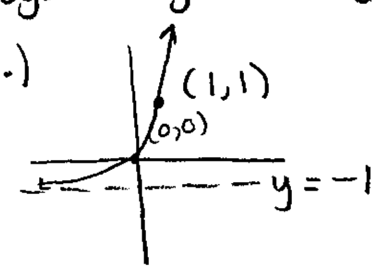


VII) 2.) $p=\frac{1}{2} \vee p=-6$ VIII 1) log 2 2) log a + log b + 3 log c − 2 log d

3.) a) $x=\frac{1}{2}$ b) $x=\frac{3}{2}$ 4.)

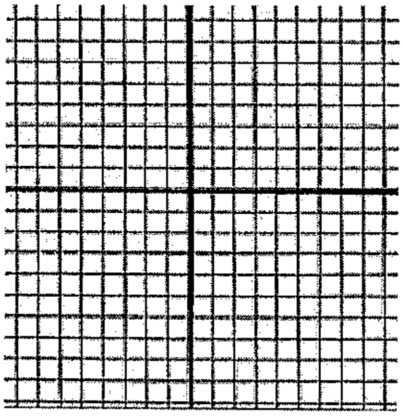


5.)

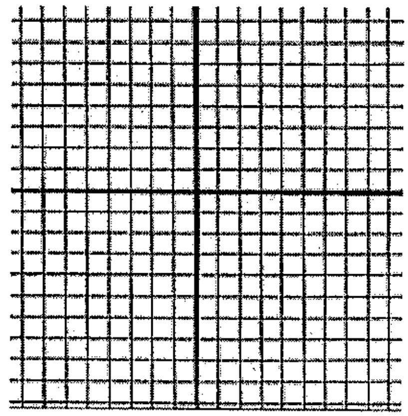


IX) Miscellaneous graphing. Graph on a coordinate system.

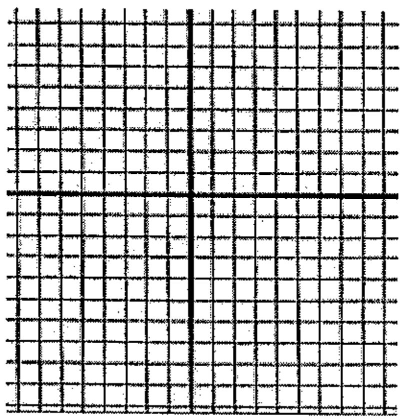
1) $x^2 + y^2 = 4$



2) $f(x) = \sqrt{x+2}$ *for* $x \geq -2$



3) $f(x) = |x - 3| - 1$



X) Polynomial functions.

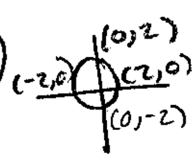
1) Find all zeros of the function: $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

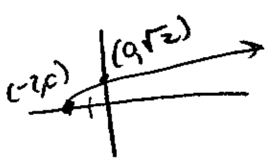
~~XI)~~ **Matrices and determinants.**

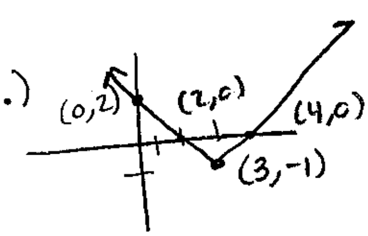
1) Evaluate the determinant of

$$\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}$$

IX)

1.) 

2.) 

3.) 

X) $x = 1$, $x = 2$, $x = -\frac{1}{2}$, $x = 4$

XII) **Sequences and series.**

1) Write the first five terms of the arithmetic sequence

$A_1 = 6, \quad A_{k+1} = A_k - 5$

2) Find the n^{th} partial sum of the arithmetic sequence

8, 20, 32, 44, . . . $\qquad n = 10$

3) Write the first five terms of the geometric sequence $a_1 = 1, \ r = 1/3$

3) Find the given sum. Round your answer to the nearest tenth.

$$\sum_{k=1}^{10} 8\left(\frac{-1}{4}\right)^{k-1}$$

XIII) **Trigonometry.**

1) Find the indicated values.

a) $\sin 30° =$ ______

b) $\cos 45° =$ ______

c) $60° =$ ______ radians

d) π radians = ______ °

e) in quadrant IV, if $\cos x = 4/5$, then $\sin x =$ ______

f) in quadrant III, if $\tan x = 3/4$, then $\cos x =$ ______

g) $\sin(2\pi/3) =$ ______

h) $\cos(\pi/2) =$ ______

i) if $\tan x = 5/6$, then $\cot x =$ ______

j) $\tan(\pi/3) =$ ______

1 a) $\frac{1}{2}$ b) $\frac{\sqrt{2}}{2}$ c) $\frac{\pi}{6}$ d) $180°$ e) $-\frac{3}{5}$ f) $-\frac{4}{5}$ g) $\frac{\sqrt{3}}{2}$

h) 0 i) $\frac{6}{5}$ j) $\sqrt{3}$